Chapter 4
Exponential and Logarithmic Functions

Section 4.1

1. $f(3) = -4(3)^2 + 5(3)$
$= -4(9) + 15$
$= -36 + 15$
$= -21$

2. $f(3x) = 4 - 2(3x)^2$
$= 4 - 2(9x^2)$
$= 4 - 18x^2$

3. $f(x) = \dfrac{x^2 - 1}{x^2 - 25}$
$x^2 - 25 \neq 0$
$(x+5)(x-5) \neq 0$
$x \neq -5, \quad x \neq 5$
Domain: $\{x \mid x \neq -5,\ x \neq 5\}$

4. $(g \circ f)(x)$ or $g(f(x))$

5. False

6. False

7. **a.** $(f \circ g)(1) = f(g(1)) = f(0) = -1$

b. $(f \circ g)(-1) = f(g(-1)) = f(0) = -1$

c. $(g \circ f)(-1) = g(f(-1)) = g(-3) = 8$

d. $(g \circ f)(0) = g(f(0)) = g(-1) = 0$

e. $(g \circ g)(-2) = g(g(-2)) = g(3) = 8$

f. $(f \circ f)(-1) = f(f(-1)) = f(-3) = -7$

8. **a.** $(f \circ g)(1) = f(g(1)) = f(0) = 5$

b. $(f \circ g)(2) = f(g(2)) = f(-3) = 11$

c. $(g \circ f)(2) = g(f(2)) = g(1) = 0$

d. $(g \circ f)(3) = g(f(3)) = g(-1) = 0$

e. $(g \circ g)(1) = g(g(1)) = g(0) = 1$

f. $(f \circ f)(3) = f(f(3)) = f(-1) = 7$

9. **a.** $g(f(-1)) = g(1) = 4$

b. $g(f(0)) = g(0) = 5$

c. $f(g(-1)) = f(3) = -1$

d. $f(g(4)) = f(2) = -2$

10. **a.** $g(f(1)) = g(-1) = 3$

b. $g(f(5)) = g(1) = 4$

c. $f(g(0)) = f(5) = 1$

d. $f(g(2)) = f(2) = -2$

11. $f(x) = 2x \qquad g(x) = 3x^2 + 1$

a. $(f \circ g)(4) = f(g(4))$
$= f\left(3(4)^2 + 1\right)$
$= f(49)$
$= 2(49)$
$= 98$

b. $(g \circ f)(2) = g(f(2))$
$= g(2 \cdot 2)$
$= g(4)$
$= 3(4)^2 + 1$
$= 48 + 1$
$= 49$

c. $(f \circ f)(1) = f(f(1))$
$= f(2(1))$
$= f(2)$
$= 2(2)$
$= 4$

d. $(g \circ g)(0) = g(g(0))$
$= g\left(3(0)^2 + 1\right)$
$= g(1)$
$= 3(1)^2 + 1$
$= 4$

12. $f(x) = 3x + 2 \qquad g(x) = 2x^2 - 1$

a. $$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(2(4)^2 - 1\right) \\ &= f(31) \\ &= 3(31) + 2 \\ &= 95\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g(3(2) + 2) \\ &= g(8) \\ &= 2(8)^2 - 1 \\ &= 128 - 1 \\ &= 127\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f\left(3(1) + 2\right) \\ &= f(5) \\ &= 3(5) + 2 \\ &= 17\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(2(0)^2 - 1\right) \\ &= g(-1) \\ &= 2(-1)^2 - 1 \\ &= 1\end{aligned}$$

13. $f(x) = 4x^2 - 3 \qquad g(x) = 3 - \frac{1}{2}x^2$

a. $$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(3 - \frac{1}{2}(4)^2\right) \\ &= f(-5) \\ &= 4(-5)^2 - 3 \\ &= 97\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g(4(2)^2 - 3) \\ &= g(13) \\ &= 3 - \frac{1}{2}(13)^2 \\ &= 3 - \frac{169}{2} \\ &= -\frac{163}{2}\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f(4(1)^2 - 3) \\ &= f(1) \\ &= 4(1)^2 - 3 \\ &= 1\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(3 - \frac{1}{2}(0)^2\right) \\ &= g(3) \\ &= 3 - \frac{1}{2}(3)^2 \\ &= 3 - \frac{9}{2} \\ &= -\frac{3}{2}\end{aligned}$$

14. $f(x) = 2x^2 \qquad g(x) = 1 - 3x^2$

a. $$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(1 - 3(4)^2\right) \\ &= f(-47) \\ &= 2(-47)^2 \\ &= 4418\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g(2(2)^2) \\ &= g(8) \\ &= 1 - 3(8)^2 \\ &= 1 - 192 \\ &= -191\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f\left(2(1)^2\right) \\ &= f(2) \\ &= 2(2)^2 \\ &= 8\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(1 - 3(0)^2\right) \\ &= g(1) \\ &= 1 - 3(1)^2 \\ &= 1 - 3 \\ &= -2\end{aligned}$$

15. $f(x)=\sqrt{x} \qquad g(x)=2x$

a. $$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(2(4)\right)\\&=f(8)\\&=\sqrt{8}\\&=2\sqrt{2}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g\left(\sqrt{2}\right)\\&=2\sqrt{2}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f\left(\sqrt{1}\right)\\&=f(1)\\&=\sqrt{1}\\&=1\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(2(0)\right)\\&=g(0)\\&=2(0)\\&=0\end{aligned}$$

16. $f(x)=\sqrt{x+1} \qquad g(x)=3x$

a. $$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(3(4)\right)\\&=f(12)\\&=\sqrt{12+1}\\&=\sqrt{13}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g\left(\sqrt{2+1}\right)\\&=g\left(\sqrt{3}\right)\\&=3\sqrt{3}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f\left(\sqrt{1+1}\right)\\&=f\left(\sqrt{2}\right)\\&=\sqrt{\sqrt{2}+1}\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(3(0)\right)\\&=g(0)\\&=3(0)\\&=0\end{aligned}$$

17. $f(x)=|\,x\,| \qquad g(x)=\dfrac{1}{x^2+1}$

a. $$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(\frac{1}{4^2+1}\right)\\&=f\left(\frac{1}{17}\right)\\&=\left|\frac{1}{17}\right|\\&=\frac{1}{17}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g\left(|\,2\,|\right)\\&=g(2)\\&=\frac{1}{2^2+1}\\&=\frac{1}{5}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f\left(|\,1\,|\right)\\&=f(1)\\&=|\,1\,|\\&=1\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(\frac{1}{0^2+1}\right)\\&=g(1)\\&=\frac{1}{1^2+1}\\&=\frac{1}{2}\end{aligned}$$

18. $f(x) = |x-2| \qquad g(x) = \dfrac{3}{x^2+2}$

a. $$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(\frac{3}{4^2+2}\right) \\ &= f\left(\frac{3}{18}\right) \\ &= f\left(\frac{1}{6}\right) \\ &= \left|\frac{1}{6}-2\right| \\ &= \left|-\frac{11}{6}\right| \\ &= \frac{11}{6}\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g(|2-2|) \\ &= g(0) \\ &= \frac{3}{0^2+2} \\ &= \frac{3}{2}\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f(|1-2|) \\ &= f(1) \\ &= |1-2| \\ &= 1\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(\frac{3}{0^2+2}\right) \\ &= g\left(\frac{3}{2}\right) \\ &= \frac{3}{\left(\frac{3}{2}\right)^2+2} \\ &= \frac{3}{\frac{17}{4}} \\ &= \frac{12}{17}\end{aligned}$$

19. $f(x) = \dfrac{3}{x+1} \qquad g(x) = \sqrt[3]{x}$

a. $$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(\sqrt[3]{4}\right) \\ &= \frac{3}{\sqrt[3]{4}+1}\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g\left(\frac{3}{2+1}\right) \\ &= g\left(\frac{3}{3}\right) \\ &= g(1) \\ &= \sqrt[3]{1} \\ &= 1\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f\left(\frac{3}{1+1}\right) \\ &= f\left(\frac{3}{2}\right) \\ &= \frac{3}{\frac{3}{2}+1} \\ &= \frac{3}{\frac{5}{2}} \\ &= \frac{6}{5}\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(\sqrt[3]{0}\right) \\ &= g(0) \\ &= \sqrt[3]{0} \\ &= 0\end{aligned}$$

20. $f(x)=x^{3/2}$ $\quad g(x)=\dfrac{2}{x+1}$

a.
$$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(\frac{2}{4+1}\right)\\&=f\left(\frac{2}{5}\right)\\&=\left(\frac{2}{5}\right)^{3/2}\\&=\sqrt{\left(\frac{2}{5}\right)^3}\\&=\sqrt{\frac{8}{125}}\\&=\frac{2\sqrt{2}}{5\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}\\&=\frac{2\sqrt{10}}{25}\end{aligned}$$

b.
$$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g\left(2^{3/2}\right)\\&=g\left(\sqrt{2^3}\right)\\&=g\left(2\sqrt{2}\right)\\&=\frac{2}{2\sqrt{2}+1}\quad\text{or}\quad\frac{4\sqrt{2}-2}{7}\end{aligned}$$

c.
$$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f\left(1^{3/2}\right)\\&=f(1)\\&=1^{3/2}\\&=1\end{aligned}$$

d.
$$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(\frac{2}{0+1}\right)\\&=g(2)\\&=\frac{2}{2+1}\\&=\frac{2}{3}\end{aligned}$$

21. The domain of g is $\{x \mid x\neq 0\}$. The domain of f is $\{x \mid x\neq 1\}$. Thus, $g(x)\neq 1$, so we solve:
$$\begin{aligned}g(x)&=1\\\frac{2}{x}&=1\\x&=2\end{aligned}$$
Thus, $x\neq 2$; so the domain of $f\circ g$ is $\{x \mid x\neq 0, x\neq 2\}$.

22. The domain of g is $\{x \mid x\neq 0\}$. The domain of f is $\{x \mid x\neq -3\}$. Thus, $g(x)\neq -3$, so we solve:
$$\begin{aligned}g(x)&=-3\\-\frac{2}{x}&=-3\\x&=\frac{2}{3}\end{aligned}$$
Thus, $x\neq \frac{2}{3}$; so the domain of $f\circ g$ is $\left\{x \,\middle|\, x\neq 0, x\neq \frac{2}{3}\right\}$.

23. The domain of g is $\{x \mid x\neq 0\}$. The domain of f is $\{x \mid x\neq 1\}$. Thus, $g(x)\neq 1$, so we solve:
$$\begin{aligned}g(x)&=1\\-\frac{4}{x}&=1\\x&=-4\end{aligned}$$
Thus, $x\neq -4$; so the domain of $f\circ g$ is $\{x \mid x\neq -4, x\neq 0\}$.

24. The domain of g is $\{x \mid x\neq 0\}$. The domain of f is $\{x \mid x\neq -3\}$. Thus, $g(x)\neq -3$, so we solve:
$$\begin{aligned}g(x)&=-3\\\frac{2}{x}&=-3\\x&=-\frac{2}{3}\end{aligned}$$
Thus, $x\neq -\frac{2}{3}$; so the domain of $f\circ g$ is $\left\{x \,\middle|\, x\neq -\frac{2}{3}, x\neq 0\right\}$.

25. The domain of g is $\{x \mid x \text{ is any real number}\}$. The domain of f is $\{x \mid x \geq 0\}$. Thus, $g(x) \geq 0$, so we solve:

$$2x + 3 \geq 0$$
$$x \geq -\frac{3}{2}$$

Thus, the domain of $f \circ g$ is $\left\{x \,\middle|\, x \geq -\frac{3}{2}\right\}$.

26. The domain of g is $\{x \mid x \leq 1\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \leq 1\}$.

27. The domain of g is $\{x \mid x \geq 1\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 1\}$.

28. The domain of g is $\{x \mid x \geq 2\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 2\}$.

29. $f(x) = 2x + 3 \qquad g(x) = 3x$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a.
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f(3x)\\ &= 2(3x) + 3\\ &= 6x + 3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b.
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g(2x + 3)\\ &= 3(2x + 3)\\ &= 6x + 9\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c.
$$\begin{aligned}(f \circ f)(x) &= f(f(x))\\ &= f(2x + 3)\\ &= 2(2x + 3) + 3\\ &= 4x + 6 + 3\\ &= 4x + 9\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d.
$$\begin{aligned}(g \circ g)(x) &= g(g(x))\\ &= g(3x)\\ &= 3(3x)\\ &= 9x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

30. $f(x) = -x \qquad g(x) = 2x - 4$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a.
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f(2x - 4)\\ &= -(2x - 4)\\ &= -2x + 4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b.
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g(-x)\\ &= 2(-x) - 4\\ &= -2x - 4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c.
$$\begin{aligned}(f \circ f)(x) &= f(f(x))\\ &= f(-x)\\ &= -(-x)\\ &= x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d.
$$\begin{aligned}(g \circ g)(x) &= g(g(x))\\ &= g(2x - 4)\\ &= 2(2x - 4) - 4\\ &= 4x - 8 - 4\\ &= 4x - 12\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

31. $f(x)=3x+1 \qquad g(x)=x^2$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2\right)\\&=3x^2+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(3x+1)\\&=(3x+1)^2\\&=9x^2+6x+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(3x+1)\\&=3(3x+1)+1\\&=9x+3+1\\&=9x+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(x^2\right)\\&=\left(x^2\right)^2\\&=x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

32. $f(x)=x+1 \qquad g(x)=x^2+4$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2+4\right)\\&=x^2+4+1\\&=x^2+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(x+1)\\&=(x+1)^2+4\\&=x^2+2x+1+4\\&=x^2+2x+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(x+1)\\&=(x+1)+1\\&=x+2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(x^2+4\right)\\&=\left(x^2+4\right)^2+4\\&=x^4+8x^2+16+4\\&=x^4+8x^2+20\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

33. $f(x)=x^2 \qquad g(x)=x^2+4$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2+4\right)\\&=\left(x^2+4\right)^2\\&=x^4+8x^2+16\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(x^2\right)\\&=\left(x^2\right)^2+4\\&=x^4+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(x^2\right)\\&=\left(x^2\right)^2\\&=x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$$= g\left(x^2+4\right)$$
$$= \left(x^2+4\right)^2+4$$
$$= x^4+8x^2+16+4$$
$$= x^4+8x^2+20$$
Domain: $\{x \mid x \text{ is any real number}\}$.

34. $f(x) = x^2+1 \qquad g(x) = 2x^2+3$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $(f \circ g)(x) = f(g(x))$
$$= f\left(2x^2+3\right)$$
$$= \left(2x^2+3\right)^2+1$$
$$= 4x^4+12x^2+9+1$$
$$= 4x^4+12x^2+10$$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(g \circ f)(x) = g(f(x))$
$$= g\left(x^2+1\right)$$
$$= 2\left(x^2+1\right)^2+3$$
$$= 2\left(x^4+2x^2+1\right)+3$$
$$= 2x^4+4x^2+2+3$$
$$= 2x^4+4x^2+5$$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \circ f)(x) = f(f(x))$
$$= f\left(x^2+1\right)$$
$$= \left(x^2+1\right)^2+1$$
$$= x^4+2x^2+1+1$$
$$= x^4+2x^2+2$$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$$= g\left(2x^2+3\right)$$
$$= 2\left(2x^2+3\right)^2+3$$
$$= 2\left(4x^4+12x^2+9\right)+3$$
$$= 8x^4+24x^2+18+3$$
$$= 8x^4+24x^2+21$$
Domain: $\{x \mid x \text{ is any real number}\}$.

35. $f(x) = \dfrac{3}{x-1} \qquad g(x) = \dfrac{2}{x}$

The domain of f is $\{x \mid x \neq 1\}$. The domain of g is $\{x \mid x \neq 0\}$.

a. $(f \circ g)(x) = f(g(x))$
$$= f\left(\frac{2}{x}\right)$$
$$= \frac{3}{\frac{2}{x}-1} = \frac{3}{\frac{2-x}{x}}$$
$$= \frac{3x}{2-x}$$
Domain $\{x \mid x \neq 0, x \neq 2\}$.

b. $(g \circ f)(x) = g(f(x))$
$$= g\left(\frac{3}{x-1}\right)$$
$$= \frac{2}{\frac{3}{x-1}}$$
$$= \frac{2(x-1)}{3}$$
Domain $\{x \mid x \neq 1\}$

c. $(f \circ f)(x) = f(f(x))$
$$= f\left(\frac{3}{x-1}\right)$$
$$= \frac{3}{\frac{3}{x-1}-1} = \frac{3}{\frac{3-(x-1)}{x-1}}$$
$$= \frac{3(x-1)}{4-x}$$
Domain $\{x \mid x \neq 1, x \neq 4\}$.

d. $(g \circ g)(x) = g(g(x)) = g\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = \frac{2x}{2} = x$

Domain $\{x \mid x \neq 0\}$.

36. $f(x) = \frac{1}{x+3}$ $\quad g(x) = -\frac{2}{x}$

The domain of *f* is $\{x \mid x \neq -3\}$. The domain of g si $\{x \mid x \neq 0\}$.

a. $(f \circ g)(x) = f(g(x))$

$$= f\left(-\frac{2}{x}\right)$$

$$= \frac{1}{-\frac{2}{x}+3} = \frac{1}{\frac{-2+3x}{x}}$$

$$= \frac{x}{-2+3x} \text{ or } \frac{x}{3x-2}$$

Domain $\left\{x \,\middle|\, x \neq 0,\, x \neq \frac{2}{3}\right\}$.

b. $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{1}{x+3}\right)$$

$$= -\frac{2}{\frac{1}{x+3}} = \frac{-2(x+3)}{1}$$

$$= -2(x+3)$$

Domain $\{x \mid x \neq -3\}$.

c. $(f \circ f)(x) = f(f(x))$

$$= f\left(\frac{1}{x+3}\right)$$

$$= \frac{1}{\frac{1}{x+3}+3} = \frac{1}{\frac{1+3x+9}{x+3}}$$

$$= \frac{x+3}{3x+10}$$

Domain $\left\{x \,\middle|\, x \neq -\frac{10}{3},\, x \neq -3\right\}$.

d. $(g \circ g)(x) = g(g(x))$

$$= g\left(-\frac{2}{x}\right)$$

$$= -\frac{2}{-\frac{2}{x}} = -\frac{2x}{-2}$$

$$= x$$

Domain $\{x \mid x \neq 0\}$.

37. $f(x) = \frac{x}{x-1}$ $\quad g(x) = -\frac{4}{x}$

The domain of *f* is $\{x \mid x \neq 1\}$. The domain of g is $\{x \mid x \neq 0\}$.

a. $(f \circ g)(x) = f(g(x))$

$$= f\left(-\frac{4}{x}\right)$$

$$= \frac{-\frac{4}{x}}{-\frac{4}{x}-1} = \frac{-\frac{4}{x}}{\frac{-4-x}{x}} = \frac{-4}{-4-x}$$

$$= \frac{4}{4+x}$$

Domain $\{x \mid x \neq -4,\, x \neq 0\}$.

b. $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x}{x-1}\right)$$

$$= -\frac{4}{\frac{x}{x-1}}$$

$$= \frac{-4(x-1)}{x}$$

Domain $\{x \mid x \neq 0,\, x \neq 1\}$.

c. $(f \circ f)(x) = f(f(x))$

$$= f\left(\frac{x}{x-1}\right)$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}}$$

$$= x$$

Domain $\{x \mid x \neq 1\}$.

d. $(g \circ g)(x) = g(g(x))$
$$= g\left(\frac{-4}{x}\right)$$
$$= -\frac{4}{-\frac{4}{x}} = \frac{-4x}{-4}$$
$$= x$$
Domain $\{x \mid x \neq 0\}$.

38. $f(x) = \frac{x}{x+3} \qquad g(x) = \frac{2}{x}$

The domain of f is $\{x \mid x \neq -3\}$. The domain of g is $\{x \mid x \neq 0\}$.

a. $(f \circ g)(x) = f(g(x))$
$$= f\left(\frac{2}{x}\right)$$
$$= \frac{\frac{2}{x}}{\frac{2}{x}+3} = \frac{\frac{2}{x}}{\frac{2+3x}{x}}$$
$$= \frac{2}{2+3x}$$
Domain $\left\{x \,\middle|\, x \neq -\frac{2}{3},\ x \neq 0\right\}$.

b. $(g \circ f)(x) = g(f(x))$
$$= g\left(\frac{x}{x+3}\right)$$
$$= \frac{2}{\frac{x}{x+3}}$$
$$= \frac{2(x+3)}{x}$$
Domain $\{x \mid x \neq -3,\ x \neq 0\}$.

c. $(f \circ f)(x) = f(f(x))$
$$= f\left(\frac{x}{x+3}\right)$$
$$= \frac{\frac{x}{x+3}}{\frac{x}{x+3}+3} = \frac{\frac{x}{x+3}}{\frac{4x+9}{x+3}}$$
$$= \frac{x}{4x+9}$$
Domain $\left\{x \,\middle|\, x \neq -3,\ x \neq -\frac{9}{4}\right\}$.

d. $(g \circ g)(x) = g(g(x)) = g\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = \frac{2x}{2} = x$

Domain $\{x \mid x \neq 0\}$.

39. $f(x) = \sqrt{x} \qquad g(x) = 2x+3$

The domain of f is $\{x \mid x \geq 0\}$. The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $(f \circ g)(x) = f(g(x)) = f(2x+3) = \sqrt{2x+3}$

Domain $\left\{x \,\middle|\, x \geq -\frac{3}{2}\right\}$.

b. $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = 2\sqrt{x}+3$

Domain $\{x \mid x \geq 0\}$.

c. $(f \circ f)(x) = f(f(x))$
$$= f\left(\sqrt{x}\right)$$
$$= \sqrt{\sqrt{x}}$$
$$= \left(x^{1/2}\right)^{1/2}$$
$$= x^{1/4}$$
$$= \sqrt[4]{x}$$
Domain $\{x \mid x \geq 0\}$.

d. $(g \circ g)(x) = g(g(x))$
$$= g(2x+3)$$
$$= 2(2x+3)+3$$
$$= 4x+6+3$$
$$= 4x+9$$
Domain $\{x \mid x \text{ is any real number}\}$.

40. $f(x)=\sqrt{x-2}$ $g(x)=1-2x$

The domain of *f* is $\{x \mid x \geq 2\}$. The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(1-2x)\\&=\sqrt{1-2x-2}\\&=\sqrt{-2x-1}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \leq -\frac{1}{2}\right\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\sqrt{x-2}\right)\\&=1-2\sqrt{x-2}\end{aligned}$$

Domain $\{x \mid x \geq 2\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(\sqrt{x-2}\right)\\&=\sqrt{\sqrt{x-2}-2}\end{aligned}$$

Now, $$\begin{aligned}\sqrt{x-2}-2&\geq 0\\ \sqrt{x-2}&\geq 2\\ x-2&\geq 4\\ x&\geq 6\end{aligned}$$

Domain $\{x \mid x \geq 6\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g(1-2x)\\&=1-2(1-2x)\\&=1-2+4x\\&=4x-1\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

41. $f(x)=x^2+1$ $g(x)=\sqrt{x-1}$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \geq 1\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(\sqrt{x-1}\right)\\&=\left(\sqrt{x-1}\right)^2+1\\&=x-1+1\\&=x\end{aligned}$$

Domain $\{x \mid x \geq 1\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(x^2+1\right)\\&=\sqrt{x^2+1-1}\\&=\sqrt{x^2}\\&=|x|\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(x^2+1\right)\\&=\left(x^2+1\right)^2+1\\&=x^4+2x^2+1+1\\&=x^4+2x^2+2\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(\sqrt{x-1}\right)\\&=\sqrt{\sqrt{x-1}-1}\end{aligned}$$

Now, $$\begin{aligned}\sqrt{x-1}-1&\geq 0\\ \sqrt{x-1}&\geq 1\\ x-1&\geq 1\\ x&\geq 2\end{aligned}$$

Domain $\{x \mid x \geq 2\}$.

42. $f(x)=x^2+4$ $g(x)=\sqrt{x-2}$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \geq 2\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(\sqrt{x-2}\right)\\&=\left(\sqrt{x-2}\right)^2+4\\&=x-2+4\\&=x+2\end{aligned}$$

Domain $\{x \mid x \geq 2\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(x^2+4\right)\\&=\sqrt{x^2+4-2}\\&=\sqrt{x^2+2}\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

c. $(f \circ f)(x) = f(f(x))$
$= f\left(x^2+4\right)$
$= \left(x^2+4\right)^2+4$
$= x^4+8x^2+16+4$
$= x^4+8x^2+20$

Domain $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$= g\left(\sqrt{x-2}\right)$
$= \sqrt{\sqrt{x-2}-2}$

Now, $\sqrt{x-2}-2 \ge 0$
$\sqrt{x-2} \ge 2$
$x-2 \ge 4$
$x \ge 6$

Domain $\{x \mid x \ge 6\}$.

43. $f(x) = ax+b \qquad g(x) = cx+d$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $(f \circ g)(x) = f(g(x))$
$= f\left(cx+d\right)$
$= a(cx+d)+b$
$= acx+ad+b$

Domain $\{x \mid x \text{ is any real number}\}$.

b. $(g \circ f)(x) = g(f(x))$
$= g\left(ax+b\right)$
$= c(ax+b)+d$
$= acx+bc+d$

Domain $\{x \mid x \text{ is any real number}\}$.

c. $(f \circ f)(x) = f(f(x))$
$= f\left(ax+b\right)$
$= a(ax+b)+b$
$= a^2x+ab+b$

Domain $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$= g\left(cx+d\right)$
$= c(cx+d)+d$
$= c^2x+cd+d$

Domain $\{x \mid x \text{ is any real number}\}$.

44. $f(x) = \dfrac{ax+b}{cx+d} \qquad g(x) = mx$

The domain of *f* is $\left\{x \,\middle|\, x \ne -\dfrac{d}{c}\right\}$. The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $(f \circ g)(x) = f(g(x))$
$= f\left(mx\right)$
$= \dfrac{a(mx)+b}{c(mx)+d}$
$= \dfrac{a\,mx+b}{c\,mx+d}$

Now, $cmx+d \ne 0$
$cmx \ne -d$
$x \ne -\dfrac{d}{cm}$

Domain $\left\{x \,\middle|\, x \ne -\dfrac{d}{cm}\right\}$.

b. $(g \circ f)(x) = g(f(x))$
$= g\left(\dfrac{ax+b}{cx+d}\right)$
$= m\left(\dfrac{ax+b}{cx+d}\right)$
$= \dfrac{amx+bm}{cx+d}$

Now, $cx+d \ne 0$
$cx \ne -d$
$x \ne -\dfrac{d}{c}$

Domain $\left\{x \,\middle|\, x \ne -\dfrac{d}{c}\right\}$.

c. $$\begin{aligned}(f\circ f)(x) &= f(f(x))\\ &= f\left(\frac{ax+b}{cx+d}\right)\\ &= \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d}\\ &= \frac{\frac{a^2x+ab+bcx+bd}{cx+d}}{\frac{acx+bc+cdx+d^2}{cx+d}}\\ &= \frac{a^2x+ab+bcx+bd}{acx+bc+cdx+d^2}\end{aligned}$$

Now, $$\begin{aligned}acx+bc+cdx+d^2 &\neq 0\\ (ac+cd)x &\neq -bc-d^2\\ x &\neq \frac{-bc-d^2}{ac+cd}\end{aligned}$$

Domain $\left\{x \,\middle|\, x\neq -\frac{d}{c},\ x\neq \frac{-bc-d^2}{ac+cd}\right\}$.

d. $$\begin{aligned}(g\circ g)(x) &= g(g(x))\\ &= g(mx)\\ &= m(mx)\\ &= m^2x\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

45. $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x\right) = 2\left(\frac{1}{2}x\right) = x$

$(g\circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x$

46. $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{4}x\right) = 4\left(\frac{1}{4}x\right) = x$

$(g\circ f)(x) = g(f(x)) = g(4x) = \frac{1}{4}(4x) = x$

47. $(f\circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

$(g\circ f)(x) = g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$

48. $(f\circ g)(x) = f(g(x)) = f(x-5) = x-5+5 = x$

$(g\circ f)(x) = g(f(x)) = g(x+5) = x+5-5 = x$

49. $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{1}{2}(x+6)\right)\\ &= 2\left(\frac{1}{2}(x+6)\right)-6\\ &= x+6-6\\ &= x\\ (g\circ f)(x) &= g(f(x))\\ &= g(2x-6)\\ &= \frac{1}{2}\left((2x-6)+6\right)\\ &= \frac{1}{2}(2x)\\ &= x\end{aligned}$$

50. $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{1}{3}(4-x)\right)\\ &= 4-3\left(\frac{1}{3}(4-x)\right)\\ &= 4-4+x\\ &= x\\ (g\circ f)(x) &= g(f(x))\\ &= g(4-3x)\\ &= \frac{1}{3}\left(4-(4-3x)\right)\\ &= \frac{1}{3}(3x)\\ &= x\end{aligned}$$

51. $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{1}{a}(x-b)\right)\\ &= a\left(\frac{1}{a}(x-b)\right)+b\\ &= x-b+b\\ &= x\\ (g\circ f)(x) &= g(f(x))\\ &= g(ax+b)\\ &= \frac{1}{a}\left((ax+b)-b\right)\\ &= \frac{1}{a}(ax)\\ &= x\end{aligned}$$

52. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

53. $H(x) = (2x+3)^4$
Answers may vary. One possibility is
$f(x) = x^4, \quad g(x) = 2x+3$

54. $H(x) = \left(1+x^2\right)^3$
Answers may vary. One possibility is
$f(x) = x^3, \quad g(x) = 1+x^2$

55. $H(x) = \sqrt{x^2+1}$
Answers may vary. One possibility is
$f(x) = \sqrt{x}, \quad g(x) = x^2+1$

56. $H(x) = \sqrt{1-x^2}$
Answers may vary. One possibility is
$f(x) = \sqrt{x}, \quad g(x) = 1-x^2$

57. $H(x) = \left|2x+1\right|$
Answers may vary. One possibility is
$f(x) = \left|x\right|, \quad g(x) = 2x+1$

58. $H(x) = \left|2x^2+3\right|$
Answer may vary. One possibility is
$f(x) = \left|x\right|, \quad g(x) = 2x^2+3$

59. $f(x) = 2x^3 - 3x^2 + 4x - 1 \qquad g(x) = 2$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2) \\ &= 2(2)^3 - 3(2)^2 + 4(2) - 1 \\ &= 16 - 12 + 8 - 1 \\ &= 11\end{aligned}$$

$(g \circ f)(x) = g(f(x)) = g\left(2x^3 - 3x^2 + 4x - 1\right) = 2$

60. $f(x) = \frac{x+1}{x-1}$

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f\left(\frac{x+1}{x-1}\right) \\ &= \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} \\ &= \frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-(x-1)}{x-1}} \\ &= \frac{\frac{2x}{x-1}}{\frac{2}{x-1}} \\ &= \frac{2x}{x-1} \cdot \frac{x-1}{2} \\ &= x\end{aligned}$$

61. $f(x) = 2x^2 + 5 \qquad g(x) = 3x + a$

$(f \circ g)(x) = f(g(x)) = f(3x+a) = 2(3x+a)^2 + 5$

When $x = 0$, $(f \circ g)(0) = 23$.

Solving:
$$\begin{aligned}2(3 \cdot 0 + a)^2 + 5 &= 23 \\ 2a^2 + 5 &= 23 \\ 2a^2 - 18 &= 0 \\ 2(a+3)(a-3) &= 0 \\ a = -3 \quad &\text{or} \quad a = 3\end{aligned}$$

62. $f(x) = 3x^2 - 7 \qquad g(x) = 2x + a$

$(f \circ g)(x) = f(g(x)) = f(2x+a) = 3(2x+a)^2 - 7$

When $x = 0$, $(f \circ g)(0) = 68$.

Solving:
$$\begin{aligned}3(2 \cdot 0 + a)^2 - 7 &= 68 \\ 3a^2 - 7 &= 68 \\ 3a^2 - 75 &= 0 \\ 3(a+5)(a-5) &= 0 \\ a = -5 \text{ or } a &= 5\end{aligned}$$

63. $S(r) = 4\pi r^2 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$$\begin{aligned} S(r(t)) &= S\left(\frac{2}{3}t^3\right) \\ &= 4\pi\left(\frac{2}{3}t^3\right)^2 \\ &= 4\pi\left(\frac{4}{9}t^6\right) \\ &= \frac{16}{9}\pi t^6 \end{aligned}$$

64. $V(r) = \frac{4}{3}\pi r^3 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$$\begin{aligned} V(r(t)) &= V\left(\frac{2}{3}t^3\right) \\ &= \frac{4}{3}\pi\left(\frac{2}{3}t^3\right)^3 \\ &= \frac{4}{3}\pi\left(\frac{8}{27}t^9\right) \\ &= \frac{32}{81}\pi t^9 \end{aligned}$$

65. $N(t) = 100t - 5t^2,\ 0 \le t \le 10$

$C(N) = 15{,}000 + 8000N$

$$\begin{aligned} C(N(t)) &= C\left(100t - 5t^2\right) \\ &= 15{,}000 + 8000\left(100t - 5t^2\right) \\ &= 15{,}000 + 800{,}000t - 40{,}000t^2 \end{aligned}$$

66. $A(r) = \pi r^2 \qquad r(t) = 200\sqrt{t}$

$A\left(r(t)\right) = A\left(200\sqrt{t}\right) = \pi\left(200\sqrt{t}\right)^2 = 40{,}000\pi t$

67. $p = -\frac{1}{4}x + 100, \quad 0 \le x \le 400$

$\frac{1}{4}x = 100 - p$

$x = 4(100 - p)$

$$\begin{aligned} C &= \frac{\sqrt{x}}{25} + 600 \\ &= \frac{\sqrt{4(100-p)}}{25} + 600 \\ &= \frac{2\sqrt{100-p}}{25} + 600, \quad 0 \le p \le 100 \end{aligned}$$

68. $p = -\frac{1}{5}x + 200, \quad 0 \le x \le 1000$

$\frac{1}{5}x = 200 - p$

$x = 5(200 - p)$

$$\begin{aligned} C &= \frac{\sqrt{x}}{10} + 400 \\ &= \frac{\sqrt{5(200-p)}}{10} + 400 \\ &= \frac{\sqrt{1000-5p}}{10} + 400, \quad 0 \le p \le 200 \end{aligned}$$

69. $V = \pi r^2 h \qquad h = 2r$

$V(r) = \pi r^2 (2r) = 2\pi r^3$

70. $V = \frac{1}{3}\pi r^2 h \quad h = 2r$

$V(r) = \frac{1}{3}\pi r^2 (2r) = \frac{2}{3}\pi r^3$

71. $f\left(x\right)$ = the number of Euros bought for x dollars;

$g\left(x\right)$ = the number of yen bought for x Euros

a. $f\left(x\right) = 0.8282x$

b. $g\left(x\right) = 139.5866x$

c.
$$\begin{aligned} g\left(f\left(x\right)\right) &= g\left(0.8282x\right) \\ &= 139.5866\left(0.8282x\right) \\ &= 115.6056221x \end{aligned}$$

d.
$$\begin{aligned} g\left(f\left(1000\right)\right) &= 115.6056221\left(1000\right) \\ &= 115{,}605.6221 \text{ yen} \end{aligned}$$

72. a. Given $C(F)=\frac{5}{9}(F-32)$ and $K(C)=C+273$, we need to find $K(C(F))$.

$$K(C(F))=\left[\frac{5}{9}(F-32)\right]+273$$
$$=\frac{5}{9}(F-32)+273$$
$$=\frac{5}{9}F-\frac{160}{9}+273$$
$$=\frac{5}{9}F+\frac{2297}{9} \quad \text{or} \quad \frac{5F+2297}{9}$$

b. $K(C(80))=\frac{5(80)+2297}{9}\approx 300$ kelvin

73. Given that *f* and *g* are odd functions, we know that $f(-x)=-f(x)$ and $g(-x)=-g(x)$ for all *x* in the domain of *f* and *g*, respectively. The composite function $(f\circ g)(x)=f(g(x))$ has the following property:

$$(f\circ g)(-x)=f(g(-x))$$
$$=f(-g(x)) \quad \text{since } g \text{ is odd}$$
$$=-f(g(x)) \quad \text{since } f \text{ is odd}$$
$$=-(f\circ g)(x)$$

Thus, $f\circ g$ is an odd function.

74. Given that *f* is odd and *g* is even, we know that $f(-x)=-f(x)$ and $g(-x)=g(x)$ for all *x* in the domain of *f* and *g*, respectively. The composite function $(f\circ g)(x)=f(g(x))$ has the following property:

$$(f\circ g)(-x)=f(g(-x))$$
$$=f(g(x)) \quad \text{since } g \text{ is even}$$
$$=(f\circ g)(x)$$

Thus, $f\circ g$ is an even function.

The composite function $(g\circ f)(x)=g(f(x))$ has the following property:

$$(g\circ f)(-x)=g(f(-x))$$
$$=g(-f(x)) \quad \text{since } f \text{ is odd}$$
$$=g(f(x)) \quad \text{since } g \text{ is even}$$
$$=(g\circ f)(x)$$

Thus, $g\circ f$ is an even function.

Section 4.2

1. The set of ordered pairs is a function because there are no ordered pairs with the same first element and different second elements.

2. The function $f(x)=x^2$ is increasing on the interval $(0,\infty)$. It is decreasing on the interval $(-\infty,0)$.

3. The function is not defined when $x^2+3x-18=0$.
Solve: $x^2+3x-18=0$
$(x+6)(x-3)=0$
$x=-6$ or $x=3$
The domain is $\{x \mid x\neq -6,\ x\neq 3\}$.

4. one-to-one

5. $y=x$

6. $[4,\infty)$

7. False. If *f* and *g* are inverse functions, then the range of *f* is the domain of *g* and the domain of *f* is the range of *g*.

8. True

9. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

10. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

11. The function is not one-to-one because there are two different inputs, 20 Hours and 50 Hours, that correspond to the same output, \$200.

12. The function is not one-to-one because there are two different inputs, John and Chuck, that correspond to the same output, Marcia.

13. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

14. The function is not one-to-one because there are two distinct inputs, 2 and -3, that correspond to the same output.

15. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

16. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

17. The function f is one-to-one because every horizontal line intersects the graph at exactly one point.

18. The function f is one-to-one because every horizontal line intersects the graph at exactly one point.

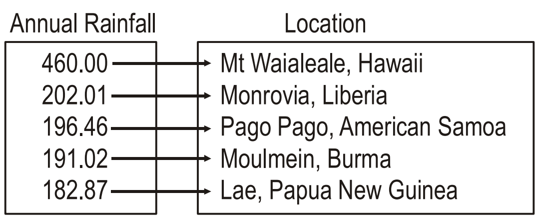
19. The function f is not one-to-one because there are horizontal lines (for example, $y = 1$) that intersect the graph at more than one point.

20. The function f is not one-to-one because there are horizontal lines (for example, $y = 1$) that intersect the graph at more than one point.

21. The function f is one-to-one because every horizontal line intersects the graph at exactly one point.

22. The function f is not one-to-one because the horizontal line $y = 2$ intersects the graph at more than one point.

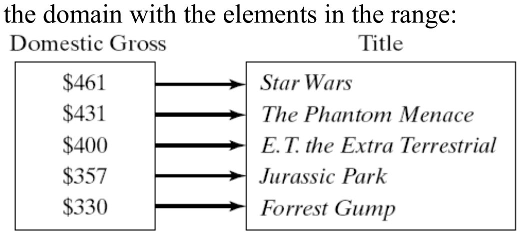
23. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {460.00, 202.01, 196.46, 191.02, 182.87}

Range: {Mt Waialeale, Monrovia, Pago Pago, Moulmein, Lea}

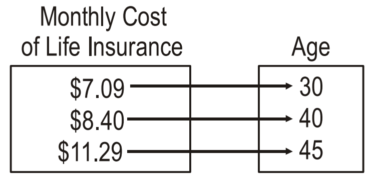
24. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {461, 431, 400, 357, 330} (in millions)

Range: {*Star Wars*, *The Phantom Menace*, *E.T. the Extra Terrestrial*, *Jurassic Park*, *Forrest Gump*}

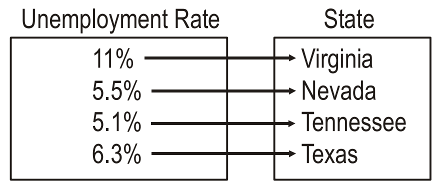
25. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {\$7.09, \$8.40, \$11.29}

Range: {30, 40, 45}

26. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {11%, 5.5%, 5.1%, 6.3%}

Range: {Virginia, Nevada, Tennessee, Texas}

27. Interchange the entries in each ordered pair: $\{(5,-3),\ (9,-2),\ (2,-1),\ (11,0),\ (-5,1)\}$

Domain: $\{5,\ 9,\ 2,\ 11,-5\}$

Range: $\{-3,-2,-1,\ 0,\ 1\}$

28. Interchange the entries in each ordered pair: $\{(2,-2),\ (6,-1),\ (8,0),\ (-3,1),\ (9,2)\}$

Domain: $\{2,\ 6,\ 8,-3,\ 9\}$

Range: $\{-2,-1,\ 0,\ 1,\ 2\}$

29. Interchange the entries in each ordered pair: $\{(1,-2),\ (2,-3),\ (0,-10),\ (9,1),\ (4,2)\}$

Domain: $\{1,\ 2,\ 0,\ 9,\ 4\}$

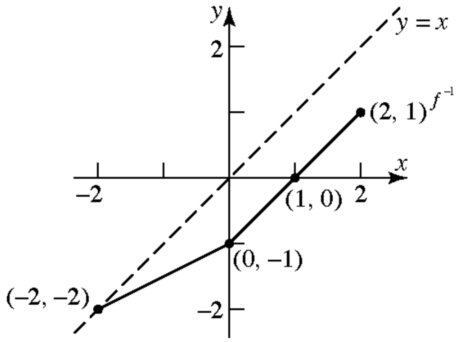
Range: $\{-2,-3,-10,\ 1,\ 2\}$

30. Interchange the entries in each ordered pair: $\{(-8,-2),\ (-1,-1),\ (0,0),\ (1,1),\ (8,2)\}$

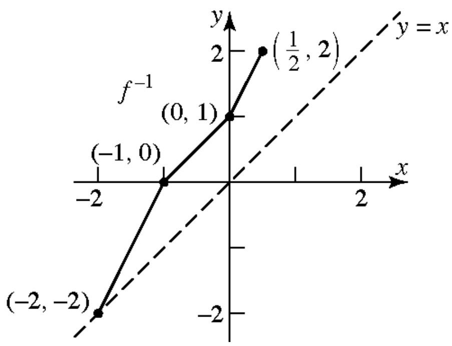
Domain: $\{-8,-1,\ 0,\ 1,\ 8\}$

Range: $\{-2,-1,\ 0,\ 1,\ 2\}$

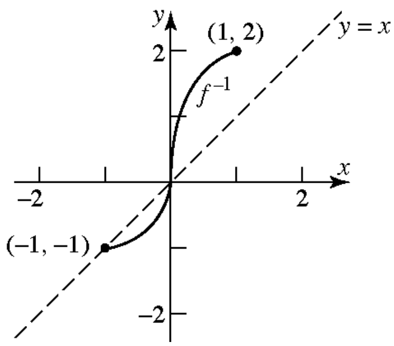
31. Graphing the inverse:



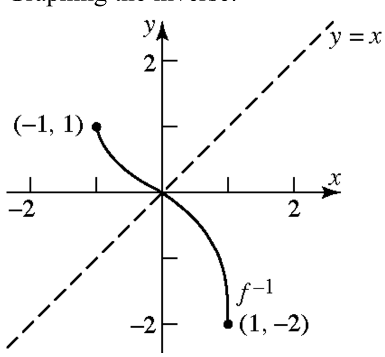
32. Graphing the inverse:



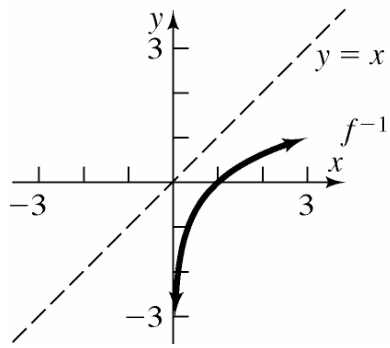
33. Graphing the inverse:



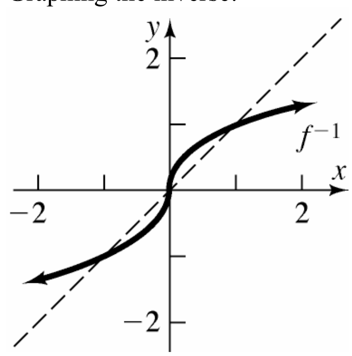
34. Graphing the inverse:



35. Graphing the inverse:



36. Graphing the inverse:



37. $f(x) = 3x + 4; \qquad g(x) = \dfrac{1}{3}(x-4)$

$$\begin{aligned}
f\left(g(x)\right) &= f\left(\frac{1}{3}(x-4)\right) \\
&= 3\left(\frac{1}{3}(x-4)\right) + 4 \\
&= (x-4) + 4 \\
&= x
\end{aligned}$$

$$\begin{aligned}
g\left(f(x)\right) &= g(3x+4) \\
&= \frac{1}{3}\left((3x+4) - 4\right) \\
&= \frac{1}{3}(3x) \\
&= x
\end{aligned}$$

Thus, f and g are inverses of each other.

38. $f(x)=3-2x; \qquad g(x)=-\frac{1}{2}(x-3)$

$$\begin{aligned} f\left(g(x)\right) &= f\left(-\frac{1}{2}(x-3)\right) \\ &= 3-2\left(-\frac{1}{2}(x-3)\right) \\ &= 3+(x-3) \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(3-2x) \\ &= -\frac{1}{2}\left((3-2x)-3\right) \\ &= -\frac{1}{2}(-2x) \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

39. $f(x)=4x-8; \qquad g(x)=\frac{x}{4}+2$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\frac{x}{4}+2\right) \\ &= 4\left(\frac{x}{4}+2\right)-8 \\ &= x+8-8 \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(4x-8) \\ &= \frac{4x-8}{4}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

40. $f(x)=2x+6; \qquad g(x)=\frac{1}{2}x-3$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\frac{1}{2}x-3\right) \\ &= 2\left(\frac{1}{2}x-3\right)+6 = x-6+6 \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(2x+6) \\ &= \frac{1}{2}(2x+6)-3 = x+3-3 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

41. $f(x)=x^3-8; \qquad g(x)=\sqrt[3]{x+8}$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\sqrt[3]{x+8}\right) \\ &= \left(\sqrt[3]{x+8}\right)^3-8 \\ &= x+8-8 \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(x^3-8) \\ &= \sqrt[3]{(x^3-8)+8} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

42. $f(x)=(x-2)^2,\ x\ge 2; \qquad g(x)=\sqrt{x}+2$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\sqrt{x}+2\right) \\ &= \left(\sqrt{x}+2-2\right)^2 \\ &= \left(\sqrt{x}\right)^2 \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g\left((x-2)^2\right) \\ &= \sqrt{(x-2)^2}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

43. $f(x)=\frac{1}{x}; \qquad g(x)=\frac{1}{x}$

$$f\left(g(x)\right) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

$$g\left(f(x)\right) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

Thus, *f* and *g* are inverses of each other.

44. $f(x)=x; \qquad g(x)=x$

$$f\left(g(x)\right) = f\left(x\right) = x$$

$$g\left(f(x)\right) = g\left(x\right) = x$$

Thus, *f* and *g* are inverses of each other.

45. $f(x)=\dfrac{2x+3}{x+4}; \qquad g(x)=\dfrac{4x-3}{2-x}$

$$f\left(g(x)\right)=f\left(\frac{4x-3}{2-x}\right)=\frac{2\left(\frac{4x-3}{2-x}\right)+3}{\frac{4x-3}{2-x}+4}$$
$$=\frac{\left(2\left(\frac{4x-3}{2-x}\right)+3\right)(2-x)}{\left(\frac{4x-3}{2-x}+4\right)(2-x)}$$
$$=\frac{2(4x-3)+3(2-x)}{4x-3+4(2-x)}$$
$$=\frac{8x-6+6-3x}{4x-3+8-4x}$$
$$=\frac{5x}{5}$$
$$=x$$

$$g\left(f(x)\right)=g\left(\frac{2x+3}{x+4}\right)$$
$$=\frac{4\left(\frac{2x+3}{x+4}\right)-3}{2-\frac{2x+3}{x+4}}$$
$$=\frac{\left(4\left(\frac{2x+3}{x+4}\right)-3\right)(x+4)}{\left(2-\frac{2x+3}{x+4}\right)(x+4)}$$
$$=\frac{4(2x+3)-3(x+4)}{2(x+4)-(2x+3)}$$
$$=\frac{8x+12-3x-12}{2x+8-2x-3}$$
$$=\frac{5x}{5}$$
$$=x$$

Thus, f and g are inverses of each other.

46. $f(x)=\dfrac{x-5}{2x+3}; \qquad g(x)=\dfrac{3x+5}{1-2x}$

$$f\left(g(x)\right)=f\left(\frac{3x+5}{1-2x}\right)$$
$$=\frac{\frac{3x+5}{1-2x}-5}{2\left(\frac{3x+5}{1-2x}\right)+3}$$
$$=\frac{\left(\frac{3x+5}{1-2x}-5\right)(1-2x)}{\left(2\left(\frac{3x+5}{1-2x}\right)+3\right)(1-2x)}$$
$$=\frac{3x+5-5(1-2x)}{2(3x+5)+3(1-2x)}$$
$$=\frac{3x+5-5+10x}{6x+10+3-6x}$$
$$=\frac{13x}{13}$$
$$=x$$

$$g\left(f(x)\right)=g\left(\frac{x-5}{2x+3}\right)$$
$$=\frac{3\left(\frac{x-5}{2x+3}\right)+5}{1-2\left(\frac{x-5}{2x+3}\right)}$$
$$=\frac{\left(3\left(\frac{x-5}{2x+3}\right)+5\right)(2x+3)}{\left(1-2\left(\frac{x-5}{2x+3}\right)\right)(2x+3)}$$
$$=\frac{3(x-5)+5(2x+3)}{1(2x+3)-2(x-5)}$$
$$=\frac{3x-15+10x+15}{2x+3-2x+10}$$
$$=\frac{13x}{13}$$
$$=x$$

Thus, f and g are inverses of each other.

47. $f(x)=3x$

$y=3x$

$x=3y$ Inverse

$y=\dfrac{x}{3}$

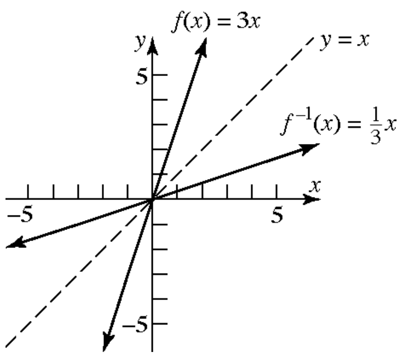
$f^{-1}(x)=\dfrac{x}{3}$

Verifying: $f\left(f^{-1}(x)\right)=f\left(\dfrac{x}{3}\right)=3\left(\dfrac{x}{3}\right)=x$

$f^{-1}\left(f(x)\right)=f^{-1}(3x)=\dfrac{3x}{3}=x$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



48. $f(x)=-4x$

$y=-4x$

$x=-4y$ Inverse

$y=\dfrac{x}{-4}$

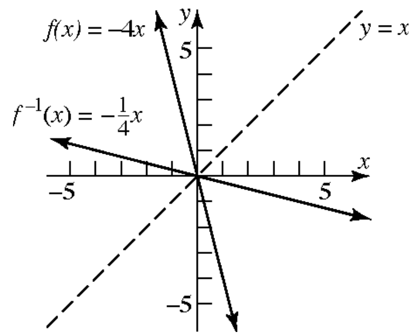
$f^{-1}(x)=-\dfrac{x}{4}$

Verifying: $f\left(f^{-1}(x)\right)=f\left(-\dfrac{x}{4}\right)=-4\left(-\dfrac{x}{4}\right)=x$

$f^{-1}\left(f(x)\right)=f^{-1}(-4x)=-\dfrac{-4x}{4}=x$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



49. $f(x)=4x+2$

$y=4x+2$

$x=4y+2$ Inverse

$4y=x-2$

$y=\dfrac{x-2}{4}$

$f^{-1}(x)=\dfrac{x-2}{4}$

Verifying:

$f\left(f^{-1}(x)\right)=f\left(\dfrac{x-2}{4}\right)=4\left(\dfrac{x-2}{4}\right)+2$

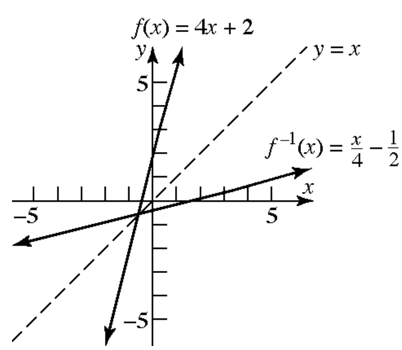
$=x-2+2=x$

$f^{-1}\left(f(x)\right)=f^{-1}\left(4x+2\right)=\dfrac{\left(4x+2\right)-2}{4}$

$=\dfrac{4x}{4}=x$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



50. $f(x) = 1 - 3x$

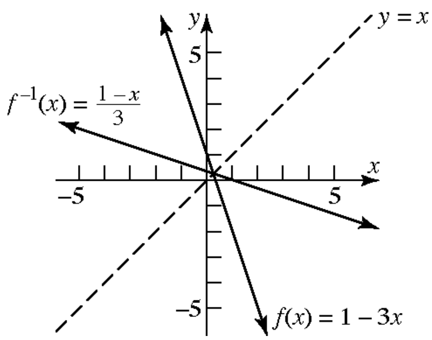
$$y = 1 - 3x$$
$$x = 1 - 3y \quad \text{Inverse}$$
$$3y = 1 - x$$
$$y = \frac{1-x}{3}$$
$$f^{-1}(x) = \frac{1-x}{3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{1-x}{3}\right) = 1 - 3\left(\frac{1-x}{3}\right)$$
$$= 1 - (1 - x) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(1-3x) = \frac{1-(1-3x)}{3} = \frac{3x}{3} = x$$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



51. $f(x) = x^3 - 1$

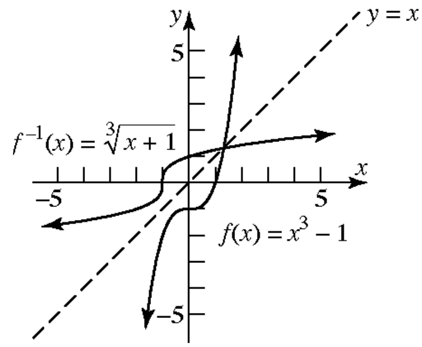
$$y = x^3 - 1$$
$$x = y^3 - 1 \quad \text{Inverse}$$
$$y^3 = x + 1$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x+1}\right) = \left(\sqrt[3]{x+1}\right)^3 - 1$$
$$= x + 1 - 1 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^3 - 1\right) = \sqrt[3]{\left(x^3 - 1\right) + 1}$$
$$= \sqrt[3]{x^3} = x$$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



52. $f(x) = x^3 + 1$

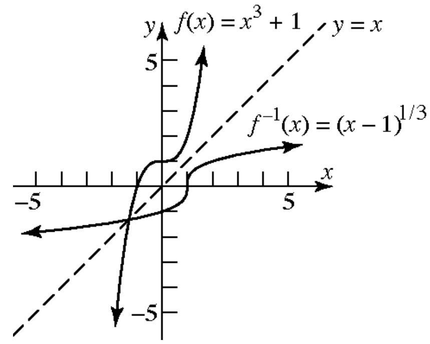
$$y = x^3 + 1$$
$$x = y^3 + 1 \quad \text{Inverse}$$
$$y^3 = x - 1$$
$$y = \sqrt[3]{x-1}$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x-1}\right) = \left(\sqrt[3]{x-1}\right)^3 + 1$$
$$= x - 1 + 1 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^3 + 1\right) = \sqrt[3]{\left(x^3 + 1\right) - 1}$$
$$= \sqrt[3]{x^3} = x$$

Domain of f = Range of f^{-1} = All real numbers

Range of f = Domain of f^{-1} = All real numbers



53. $f(x) = x^2 + 4, \ x \ge 0$

$y = x^2 + 4 \quad x \ge 0$

$x = y^2 + 4 \quad y \ge 0 \quad$ Inverse

$y^2 = x - 4 \quad y \ge 0$

$y = \sqrt{x-4}$

$f^{-1}(x) = \sqrt{x-4}$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt{x-4}\right)$

$= \left(\sqrt{x-4}\right)^2 + 4$

$= x - 4 + 4 = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(x^2 + 4\right)$

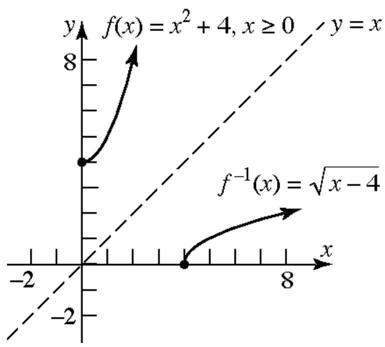
$= \sqrt{\left(x^2+4\right) - 4}$

$= \sqrt{x^2} = |\,x\,|$

$= x, \ x \ge 0$

Domain of f = Range of f^{-1} = $\{x \mid x \ge 0\}$ or $[0, \infty)$

Range of f = Domain of f^{-1} = $\{x \mid x \ge 4\}$ or $[4, \infty)$



54. $f(x) = x^2 + 9, \ x \ge 0$

$y = x^2 + 9 \quad x \ge 0$

$x = y^2 + 9 \quad y \ge 0 \quad$ Inverse

$y^2 = x - 9 \quad y \ge 0$

$y = \sqrt{x-9}$

$f^{-1}(x) = \sqrt{x-9}$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt{x-9}\right)$

$= \left(\sqrt{x-9}\right)^2 + 9$

$= x - 9 + 9$

$= x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(x^2 + 9\right)$

$= \sqrt{\left(x^2+9\right) - 9}$

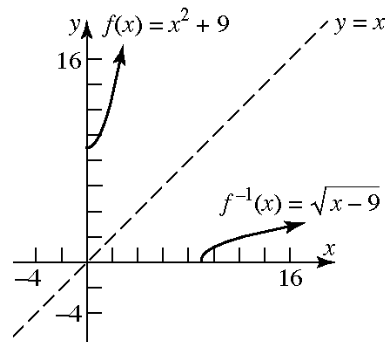
$= \sqrt{x^2}$

$= |\,x\,|$

$= x, \ x \ge 0$

Domain of f = Range of f^{-1} = $\{x \mid x \ge 0\}$ or $[0, \infty)$

Range of f = Domain of f^{-1} = $\{x \mid x \ge 9\}$ or $[9, \infty)$



55. $f(x) = \dfrac{4}{x}$

$y = \dfrac{4}{x}$

$x = \dfrac{4}{y} \quad$ Inverse

$xy = 4$

$y = \dfrac{4}{x}$

$f^{-1}(x) = \dfrac{4}{x}$

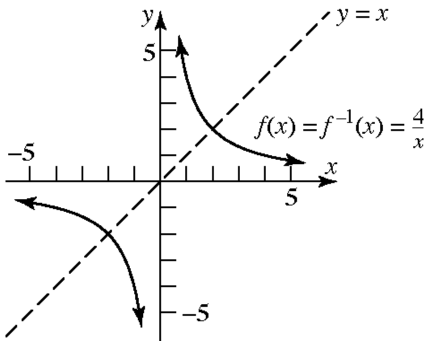
Verifying:

$f\left(f^{-1}(x)\right) = f\left(\dfrac{4}{x}\right) = \dfrac{4}{\frac{4}{x}} = 4 \cdot \left(\dfrac{x}{4}\right) = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(\dfrac{4}{x}\right) = \dfrac{4}{\frac{4}{x}} = 4 \cdot \left(\dfrac{x}{4}\right) = x$

Domain of f = Range of f^{-1}
= All real numbers except 0.

Range of f = Domain of f^{-1}
= All real numbers except 0.



56. $f(x) = -\dfrac{3}{x}$

$y = -\dfrac{3}{x}$

$x = -\dfrac{3}{y}$ Inverse

$xy = -3$

$y = -\dfrac{3}{x}$

$f^{-1}(x) = -\dfrac{3}{x}$

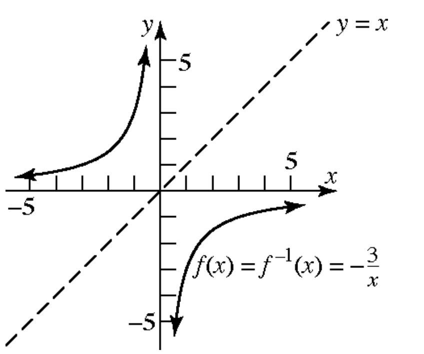
Verifying:

$f\left(f^{-1}(x)\right) = f\left(-\dfrac{3}{x}\right) = -\dfrac{3}{-\dfrac{3}{x}} = -3\cdot\left(-\dfrac{x}{3}\right) = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(-\dfrac{3}{x}\right) = -\dfrac{3}{-\dfrac{3}{x}} = -3\cdot\left(-\dfrac{x}{3}\right) = x$

Domain of f = Range of f^{-1}
= All real numbers except 0.

Range of f = Domain of f^{-1}
= All real numbers except 0.



57. $f(x) = \dfrac{1}{x-2}$

$y = \dfrac{1}{x-2}$

$x = \dfrac{1}{y-2}$ Inverse

$xy - 2x = 1$

$xy = 2x + 1$

$y = \dfrac{2x+1}{x}$

$f^{-1}(x) = \dfrac{2x+1}{x}$

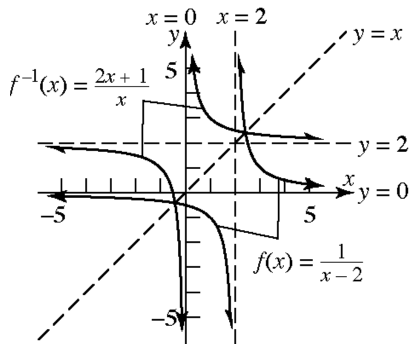
Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{2x+1}{x}\right) = \frac{1}{\frac{2x+1}{x}-2} = \frac{1\cdot x}{\left(\frac{2x+1}{x}-2\right)x} = \frac{x}{2x+1-2x} = \frac{x}{1} = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{1}{x-2}\right) = \frac{2\left(\frac{1}{x-2}\right)+1}{\frac{1}{x-2}} = \frac{\left(2\left(\frac{1}{x-2}\right)+1\right)(x-2)}{\left(\frac{1}{x-2}\right)(x-2)} = \frac{2+(x-2)}{1} = \frac{x}{1} = x$$

Domain of f = Range of f^{-1}
= All real numbers except 0.

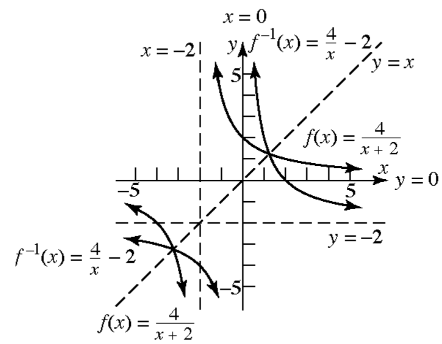
Range of f = Domain of f^{-1}
= All real numbers except 2.



58.
$$f(x) = \frac{4}{x+2}$$
$$y = \frac{4}{x+2}$$
$$x = \frac{4}{y+2} \quad \text{Inverse}$$
$$x(y+2) = 4$$
$$xy + 2x = 4$$
$$xy = 4 - 2x$$
$$y = \frac{4-2x}{x}$$
$$f^{-1}(x) = \frac{4-2x}{x}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{4-2x}{x}\right)$$
$$= \frac{4}{\frac{4-2x}{x}+2} = \frac{4\cdot x}{\left(\frac{4-2x}{x}+2\right)x}$$
$$= \frac{4x}{4-2x+2x} = \frac{4x}{4} = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{4}{x+2}\right)$$
$$= \frac{4-2\left(\frac{4}{x+2}\right)}{\frac{4}{x+2}}$$
$$= \frac{\left(4-2\left(\frac{4}{x+2}\right)\right)(x+2)}{\left(\frac{4}{x+2}\right)(x+2)}$$
$$= \frac{4(x+2)-2(4)}{4}$$
$$= \frac{4x+8-8}{4} = \frac{4x}{4} = x$$

Domain of f = Range of f^{-1}
= All real numbers except -2.
Range of f = Domain of f^{-1}
= All real numbers except 0.



59.
$$f(x) = \frac{2}{3+x}$$
$$y = \frac{2}{3+x}$$
$$x = \frac{2}{3+y} \quad \text{Inverse}$$
$$x(3+y) = 2$$
$$3x + xy = 2$$
$$xy = 2 - 3x$$
$$y = \frac{2-3x}{x}$$
$$f^{-1}(x) = \frac{2-3x}{x}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{2-3x}{x}\right)$$
$$= \frac{2}{3+\frac{2-3x}{x}} = \frac{2\cdot x}{\left(3+\frac{2-3x}{x}\right)x}$$
$$= \frac{2x}{3x+2-3x} = \frac{2x}{2}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2}{3+x}\right)$$
$$= \frac{2-3\left(\frac{2}{3+x}\right)}{\frac{2}{3+x}}$$
$$= \frac{\left(2-3\left(\frac{2}{3+x}\right)\right)(3+x)}{\left(\frac{2}{3+x}\right)(3+x)}$$
$$= \frac{2(3+x)-3(2)}{2} = \frac{6+2x-6}{2}$$
$$= \frac{2x}{2} = x$$

Domain of f = Range of f^{-1}
= All real numbers except -3.
Range of f = Domain of f^{-1}
= All real numbers except 0.

60. $f(x) = \frac{4}{2-x}$

$$y = \frac{4}{2-x}$$
$$x = \frac{4}{2-y} \quad \text{Inverse}$$
$$x(2-y) = 4$$
$$2x - xy = 4$$
$$xy = 2x-4$$
$$y = \frac{2x-4}{x}$$
$$f^{-1}(x) = \frac{2x-4}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{2x-4}{x}\right)$$
$$= \frac{4}{2-\frac{2x-4}{x}} = \frac{4\cdot x}{\left(2-\frac{2x-4}{x}\right)x}$$
$$= \frac{4x}{2x-(2x-4)} = \frac{4x}{4}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{4}{2-x}\right)$$
$$= \frac{2\left(\frac{4}{2-x}\right)-4}{\frac{4}{2-x}}$$
$$= \frac{\left(2\left(\frac{4}{2-x}\right)-4\right)(2-x)}{\left(\frac{4}{2-x}\right)(2-x)}$$
$$= \frac{2(4)-4(2-x)}{4} = \frac{8-8+4x}{4} = \frac{4x}{4}$$
$$= x$$

Domain of f = Range of f^{-1}
= All real numbers except 2.
Range of f = Domain of f^{-1}
= All real numbers except 0.

61. $f(x) = \frac{3x}{x+2}$

$$y = \frac{3x}{x+2}$$
$$x = \frac{3y}{y+2} \quad \text{Inverse}$$
$$x(y+2) = 3y$$
$$xy + 2x = 3y$$
$$xy - 3y = -2x$$
$$y(x-3) = -2x$$
$$y = \frac{-2x}{x-3}$$
$$f^{-1}(x) = \frac{-2x}{x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{-2x}{x-3}\right) = \frac{3\left(\frac{-2x}{x-3}\right)}{\frac{-2x}{x-3}+2}$$
$$= \frac{\left(3\left(\frac{-2x}{x-3}\right)\right)(x-3)}{\left(\frac{-2x}{x-3}+2\right)(x-3)}$$
$$= \frac{-6x}{-2x+2x-6} = \frac{-6x}{-6}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x}{x+2}\right) = \frac{-2\left(\frac{3x}{x+2}\right)}{\frac{3x}{x+2}-3}$$
$$= \frac{\left(-2\left(\frac{3x}{x+2}\right)\right)(x+2)}{\left(\frac{3x}{x+2}-3\right)(x+2)}$$
$$= \frac{-6x}{3x-3x-6} = \frac{-6x}{-6}$$
$$= x$$

Domain of f = Range of f^{-1}
= All real numbers except -2.
Range of f = Domain of f^{-1}
= All real numbers except 3.

62. $f(x) = \frac{-2x}{x-1}$
$$y = \frac{-2x}{x-1}$$
$$x = \frac{-2y}{y-1} \quad \text{Inverse}$$
$$x(y-1) = -2y$$
$$xy + -x = -2y$$
$$xy + 2y = x$$
$$y(x+2) = x$$
$$y = \frac{x}{x+2}$$
$$f^{-1}(x) = \frac{x}{x+2}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{x}{x+2}\right) = \frac{-2\left(\frac{x}{x+2}\right)}{\frac{x}{x+2}-1}$$
$$= \frac{\left(-2\left(\frac{x}{x+2}\right)\right)(x+2)}{\left(\frac{x}{x+2}-1\right)(x+2)}$$
$$= \frac{-2x}{x-(x+2)}$$
$$= \frac{-2x}{-2}$$
$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-2x}{x-1}\right) = \frac{\frac{-2x}{x-1}}{\frac{-2x}{x-1}+2}$$
$$= \frac{\left(\frac{-2x}{x-1}\right)(x-1)}{\left(\frac{-2x}{x-1}+2\right)(x-1)}$$
$$= \frac{-2x}{-2x+2x-2}$$
$$= \frac{-2x}{-2}$$
$$= x$$

Domain of f = Range of f^{-1}
= All real numbers except 1.
Range of f = Domain of f^{-1}
= All real numbers except -2.

63. $f(x) = \frac{2x}{3x-1}$
$$y = \frac{2x}{3x-1}$$
$$x = \frac{2y}{3y-1} \quad \text{Inverse}$$
$$3xy - x = 2y$$
$$3xy - 2y = x$$
$$y(3x-2) = x$$
$$y = \frac{x}{3x-2}$$
$$f^{-1}(x) = \frac{x}{3x-2}$$

Verifying:
$$f\left(f^{-1}(x)\right) = f\left(\frac{x}{3x-2}\right) = \frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right)-1}$$
$$= \frac{\left(2\left(\frac{x}{3x-2}\right)\right)(3x-2)}{\left(3\left(\frac{x}{3x-2}\right)-1\right)(3x-2)}$$
$$= \frac{2x}{3x-(3x-2)} = \frac{2x}{2}$$
$$= x$$

$$f^{-1}\left(f(x)\right) = f\left(\frac{2x}{3x-1}\right) = \frac{\frac{2x}{3x-1}}{3\left(\frac{2x}{3x-1}\right)-2}$$

$$= \frac{\left(\frac{2x}{3x-1}\right)(3x-1)}{\left(3\left(\frac{2x}{3x-1}\right)-2\right)(3x-1)}$$

$$= \frac{2x}{3(2x)-2(3x-1)}$$

$$= \frac{2x}{6x-6x+2} = \frac{2x}{2}$$

$$= x$$

Domain of f = Range of f^{-1}

= All real numbers except $1/3$.

Range of f = Domain of f^{-1}

= All real numbers except $2/3$.

64. $$f(x) = \frac{3x+1}{-x}$$

$$y = \frac{3x+1}{-x}$$

$$x = \frac{3y+1}{-y} \quad \text{Inverse}$$

$$-xy = 3y+1$$

$$-xy-3y = 1$$

$$-y(x+3) = 1$$

$$y = \frac{-1}{x+3}$$

$$f^{-1}(x) = \frac{-1}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{x}{x+2}\right)$$

$$= \frac{3\left(-\frac{1}{x+3}\right)+1}{-\left(-\frac{1}{x+3}\right)} = \frac{\frac{-3}{x+3}+1}{\frac{1}{x+3}}$$

$$= \left(\frac{-3}{x+3}+1\right)\cdot\frac{x+3}{1}$$

$$= -3+(x+3)$$

$$= x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{3x+1}{-x}\right)$$

$$= \frac{1}{-\left(\frac{3x+1}{-x}\right)-3} = \frac{1}{\frac{3x+1}{x}-3}$$

$$= \frac{1\cdot x}{\left(\frac{3x+1}{x}-3\right)x} = \frac{x}{3x+1-3x}$$

$$= \frac{x}{1} = x$$

Domain of f = Range of f^{-1}

= All real numbers except 0.

Range of f = Domain of f^{-1}

= All real numbers except -3.

65. $$f(x) = \frac{3x+4}{2x-3}$$

$$y = \frac{3x+4}{2x-3}$$

$$x = \frac{3y+4}{2y-3} \quad \text{Inverse}$$

$$x(2y-3) = 3y+4$$

$$2xy-3x = 3y+4$$

$$2xy-3y = 3x+4$$

$$y(2x-3) = 3x+4$$

$$y = \frac{3x+4}{2x-3}$$

$$f^{-1}(x) = \frac{3x+4}{2x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{x}{x+2}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$

$$= \frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$

$$= \frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$

$$= \frac{9x+12+8x-12}{6x+8-6x+9} = \frac{17x}{17}$$

$$= x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$
$$= \frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$
$$= \frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$
$$= \frac{9x+12+8x-12}{6x+8-6x+9} = \frac{17x}{17}$$
$$= x$$

Domain of f = Range of f^{-1}

= All real numbers except $3/2$.

Range of f = Domain of f^{-1}

= All real numbers except $3/2$.

66. $$f(x) = \frac{2x-3}{x+4}$$
$$y = \frac{2x-3}{x+4}$$
$$x = \frac{2y-3}{y+4} \quad \text{Inverse}$$
$$x(y+4) = 2y-3$$
$$xy+4x = 2y-3$$
$$xy-2y = -4x-3$$
$$y(x-2) = -(4x+3)$$
$$y = \frac{-(4x+3)}{x-2} = \frac{4x+3}{2-x}$$
$$f^{-1}(x) = \frac{4x+3}{2-x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{4x+3}{2-x}\right)$$
$$= \frac{2\left(\frac{4x+3}{2-x}\right)-3}{\frac{4x+3}{2-x}+4} = \frac{2(4x+3)-3(2-x)}{4x+3+4(2-x)}$$
$$= \frac{8x+6-6+3x}{4x+3+8-4x} = \frac{11x}{11}$$
$$= x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{2x-3}{x+4}\right)$$
$$= \frac{-4\left(\frac{2x-3}{x+4}\right)-3}{\frac{2x-3}{x+4}-2}$$
$$= \frac{-4(2x-3)-3(x+4)}{2x-3-2(x+4)}$$
$$= \frac{-8x+12-3x-12}{2x-3-2x-8}$$
$$= \frac{-11x}{-11}$$
$$= x$$

Domain of f = Range of f^{-1}

= All real numbers except -4.

Range of f = Domain of f^{-1}

= All real numbers except 2.

67. $$f(x) = \frac{2x+3}{x+2}$$
$$y = \frac{2x+3}{x+2}$$
$$x = \frac{2y+3}{y+2} \quad \text{Inverse}$$
$$xy+2x = 2y+3$$
$$xy-2y = -2x+3$$
$$y(x-2) = -2x+3$$
$$y = \frac{-2x+3}{x-2}$$
$$f^{-1}(x) = \frac{-2x+3}{x-2}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{-2x+3}{x-2}\right) = \frac{2\left(\frac{-2x+3}{x-2}\right)+3}{\frac{-2x+3}{x-2}+2}$$
$$= \frac{\left(2\left(\frac{-2x+3}{x-2}\right)+3\right)(x-2)}{\left(\frac{-2x+3}{x-2}+2\right)(x-2)}$$
$$= \frac{2(-2x+3)+3(x-2)}{-2x+3+2(x-2)}$$
$$= \frac{-4x+6+3x-6}{-2x+3+2x-4} = \frac{-x}{-1} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+3}{x+2}\right) = \frac{-2\left(\frac{2x+3}{x+2}\right)+3}{\frac{2x+3}{x+2}-2}$$

$$= \frac{\left(-2\left(\frac{2x+3}{x+2}\right)+3\right)(x+2)}{\left(\frac{2x+3}{x+2}-2\right)(x+2)}$$

$$= \frac{-2(2x+3)+3(x+2)}{2x+3-2(x+2)}$$

$$= \frac{-4x-6+3x+6}{2x+3-2x-4} = \frac{-x}{-1} = x$$

Domain of f = Range of f^{-1}
= All real numbers except -2.

Range of f = Domain of f^{-1}
= All real numbers except 2.

68.

$$f(x) = \frac{-3x-4}{x-2}$$

$$y = \frac{-3x-4}{x-2}$$

$$x = \frac{-3y-4}{y-2} \quad \text{Inverse}$$

$$x(y-2) = -3y-4$$

$$xy-2x = -3y-4$$

$$xy+3y = 2x-4$$

$$y(x+3) = 2x-4$$

$$y = \frac{2x-4}{x+3}$$

$$f^{-1}(x) = \frac{2x-4}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{2x-4}{x+3}\right)$$

$$= \frac{-3\left(\frac{2x-4}{x+3}\right)-4}{\frac{2x-4}{x+3}-2}$$

$$= \frac{-3(2x-4)-4(x+3)}{2x-4-2(x+3)}$$

$$= \frac{-6x+12-4x-12}{2x-4-2x-6}$$

$$= \frac{-10x}{-10}$$

$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{-3x-4}{x-2}\right)$$

$$= \frac{2\left(\frac{-3x-4}{x-2}\right)-4}{\frac{-3x-4}{x-2}+3}$$

$$= \frac{2(-3x-4)-4(x-2)}{-3x-4+3(x-2)}$$

$$= \frac{-6x-8-4x+8}{-3x-4+3x-6}$$

$$= \frac{-10x}{-10}$$

$$= x$$

Domain of f = Range of f^{-1}
= All real numbers except 2.

Range of f = Domain of f^{-1}
= All real numbers except -3.

69.

$$f(x) = \frac{x^2-4}{2x^2}, \; x > 0$$

$$y = \frac{x^2-4}{2x^2}, \quad x > 0$$

$$x = \frac{y^2-4}{2y^2}, \quad y > 0 \quad \text{Inverse}$$

$$2xy^2 = y^2-4, \quad x < \frac{1}{2}$$

$$2xy^2 - y^2 = -4, \quad x < \frac{1}{2}$$

$$y^2(2x-1) = -4, \quad x < \frac{1}{2}$$

$$y^2(1-2x) = 4, \quad x < \frac{1}{2}$$

$$y^2 = \frac{4}{1-2x}, \quad x < \frac{1}{2}$$

$$y = \sqrt{\frac{4}{1-2x}}, \quad x < \frac{1}{2}$$

$$y = \frac{2}{\sqrt{1-2x}}, \quad x < \frac{1}{2}$$

$$f^{-1}(x) = \frac{2}{\sqrt{1-2x}}, \quad x < \frac{1}{2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2}{\sqrt{1-2x}}\right)$$

$$=\frac{\left(\frac{2}{\sqrt{1-2x}}\right)^2-4}{2\left(\frac{2}{\sqrt{1-2x}}\right)^2}=\frac{\frac{4}{1-2x}-4}{2\left(\frac{4}{1-2x}\right)}$$

$$=\frac{\left(\frac{4}{1-2x}-4\right)(1-2x)}{\left(2\left(\frac{4}{1-2x}\right)\right)(1-2x)}$$

$$=\frac{4-4(1-2x)}{2(4)}=\frac{4-4+8x}{8}$$

$$=\frac{8x}{8}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{x^2-4}{2x^2}\right)$$

$$=\frac{2}{\sqrt{1-2\left(\frac{x^2-4}{2x^2}\right)}}=\frac{2}{\sqrt{1-\frac{x^2-4}{x^2}}}$$

$$=\frac{2}{\sqrt{1-1+\frac{4}{x^2}}}=\frac{2}{\sqrt{\frac{4}{x^2}}}$$

$$=\frac{2}{\frac{2}{|x|}}=2\cdot\frac{|x|}{2}=|x|$$

$$=x,\qquad x>0$$

Domain of f = Range of f^{-1}

$= \{x \mid x>0\}$ or $(0,\ \infty)$

Range of f = Domain of f^{-1}

$=\left\{x \,\middle|\, x<\frac{1}{2}\right\}$ or $\left(-\infty,\ \frac{1}{2}\right)$

70. $f(x)=\dfrac{x^2+3}{3x^2},\qquad x>0$

$$y=\frac{x^2+3}{3x^2},\qquad x>0$$

$$x=\frac{y^2+3}{3y^2},\qquad y>0\quad \text{Inverse}$$

$$3xy^2=y^2+3,\qquad x>\frac{1}{3}$$

$$3xy^2-y^2=3,\qquad x>\frac{1}{3}$$

$$y^2\left(3x-1\right)=3,\qquad x>\frac{1}{3}$$

$$y^2=\frac{3}{3x-1},\qquad x>\frac{1}{3}$$

$$y=\sqrt{\frac{3}{3x-1}},\quad x>\frac{1}{3}$$

$$f^{-1}\left(x\right)=\sqrt{\frac{3}{3x-1}},\quad x>\frac{1}{3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\sqrt{\frac{3}{3x-1}}\right)$$

$$=\frac{\left(\sqrt{\frac{3}{3x-1}}\right)^2+3}{3\left(\sqrt{\frac{3}{3x-1}}\right)^2}=\frac{\frac{3}{3x-1}+3}{3\left(\frac{3}{3x-1}\right)}$$

$$=\frac{\left(\frac{3}{3x-1}+3\right)(3x-1)}{\left(3\left(\frac{3}{3x-1}\right)\right)(3x-1)}=\frac{3+3(3x-1)}{3(3)}$$

$$=\frac{3+9x-3}{9}=\frac{9x}{9}$$

$$=x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^2+3}{3x^2}\right)$$
$$= \sqrt{\frac{3}{3\left(\frac{x^2+3}{3x^2}\right)-1}} = \sqrt{\frac{3}{\frac{x^2+3}{x^2}-1}}$$
$$= \sqrt{\frac{3}{1+\frac{3}{x^2}-1}} = \sqrt{\frac{3}{\frac{3}{x^2}}}$$
$$= \sqrt{(3)\left(\frac{x^2}{3}\right)} = \sqrt{x^2}$$
$$= |x|$$
$$= x, \qquad x > 0$$

Domain of f = Range of f^{-1}
$= \{x \mid x > 0\}$ or $(0, \infty)$

Range of f = Domain of f^{-1}
$= \left\{x \,\middle|\, x > \frac{1}{3}\right\}$ or $\left(\frac{1}{3}, \infty\right)$

71. a. Because the ordered pair $(-1, 0)$ is on the graph, $f(-1) = 0$.

b. Because the ordered pair $(1, 2)$ is on the graph, $f(1) = 2$.

c. Because the ordered pair $(0, 1)$ is on the graph, $f^{-1}(1) = 0$.

d. Because the ordered pair $(1, 2)$ is on the graph, $f^{-1}(2) = 1$.

72. a. Because the ordered pair $\left(2, \frac{1}{2}\right)$ is on the graph, $f(2) = \frac{1}{2}$.

b. Because the ordered pair $(1, 0)$ is on the graph, $f(1) = 0$.

c. Because the ordered pair $(1, 0)$ is on the graph, $f^{-1}(0) = 1$.

d. Because the ordered pair $(0, -1)$ is on the graph, $f^{-1}(-1) = 0$.

73. Since $f(7) = 13$, we have $f^{-1}(13) = 7$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

74. Since $g(-5) = 3$, we have $g^{-1}(3) = -5$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

75. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:
Domain: $[-2, \infty)$ Range: $[5, \infty]$

76. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:
Domain: $[5, \infty)$ Range: $[0, \infty)$

77. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $[0, \infty)$ Range: all real numbers

78. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $(0, 8)$ Range: $[0, 15]$

79. Since $f(x)$ is increasing on the interval $(0, 5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is increasing on the interval $(f(0), f(5))$.

80. Since $f(x)$ is decreasing on the interval $(0, 5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is decreasing on the interval $(f(0), f(5))$.

81. $f(x) = mx + b, \quad m \neq 0$

$$y = mx + b$$
$$x = my + b \quad \text{Inverse}$$
$$x - b = my$$
$$y = \frac{x-b}{m}$$
$$f^{-1}(x) = \frac{x-b}{m}, \quad m \neq 0$$

82. $f(x) = \sqrt{r^2 - x^2}, \quad 0 \le x \le r$

$$y = \sqrt{r^2 - x^2}$$
$$x = \sqrt{r^2 - y^2} \quad \text{Inverse}$$
$$x^2 = r^2 - y^2$$
$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$
$$f^{-1}(x) = \sqrt{r^2 - x^2}, \quad 0 \le x \le r$$

83. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of f^{-1} lies in quadrant I, both coordinates of (a,b) are positive, which means that both coordinates of (b,a) are positive. Thus, the graph of f^{-1} must lie in quadrant I.

84. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of *f* lies in quadrant II, *a* must be negative and *b* must be positive. Thus, (b,a) must be a point in quadrant IV, which means the graph of f^{-1} lies in quadrant IV.

85. Answers may vary. One possibility follows: $f(x) = |x|, x \ge 0$ is one-to-one.

Thus, $f(x) = x, x \ge 0$

$$y = x, x \ge 0$$
$$f^{-1}(x) = x, x \ge 0$$

86. Answers may vary. One possibility follows: $f(x) = x^4, x \ge 0$ is one-to-one.

Thus, $f(x) = x^4, \quad x \ge 0$

$$y = x^4, \quad x \ge 0$$
$$x = y^4 \quad \text{Inverse}$$
$$y = \sqrt[4]{x}, \quad x \ge 0$$
$$f^{-1}(x) = \sqrt[4]{x}, \quad x \ge 0$$

87. a.

$$d = 6.97r - 90.39$$
$$d + 90.39 = 6.97r$$
$$\frac{d + 90.39}{6.97} = r$$

Therefore, we would write

$$r(d) = \frac{d + 90.39}{6.97}$$

b. $r(d(r)) = \frac{(6.97r - 90.39) + 90.39}{6.97}$

$$= \frac{6.97r + 90.39 - 90.39}{6.97} = \frac{6.97r}{6.97}$$
$$= r$$

$$d(r(d)) = 6.97\left(\frac{d + 90.39}{6.97}\right) - 90.39$$
$$= d + 90.39 - 90.39$$
$$= d$$

c. $r(300) = \frac{300 + 90.39}{6.97} \approx 56.01$

If the distance required to stop was 300 feet, the speed of the car was roughly 56 miles per hour.

88. a.

$$H(C) = 2.15C - 10.53$$
$$H = 2.15C - 10.53$$
$$H + 10.53 = 2.15C$$
$$\frac{H + 10.53}{2.15} = C$$
$$C(H) = \frac{H + 10.53}{2.15}$$

b. $H(C(H)) = 2.15\left(\frac{H+10.53}{2.15}\right) - 10.53$
$= H + 10.53 - 10.53$
$= H$

$C(H(C)) = \frac{(2.15C - 10.53) + 10.53}{2.15}$
$= \frac{2.15C - 10.53 + 10.53}{2.15}$
$= \frac{2.15C}{2.15} = C$

c. $C(26) = \frac{26 + 10.53}{2.15} \approx 16.99$

The head circumference of a child who is 26 inches tall is about 17 inches.

89. a. 6 feet = 72 inches

$W(72) = 50 + 2.3(72 - 60)$
$= 50 + 2.3(12)$
$= 50 + 27.6$
$= 77.6$

The ideal weight of a 6-foot male is 77.6 kilograms.

b. $W = 50 + 2.3(h - 60)$
$W - 50 = 2.3h - 138$
$W + 88 = 2.3h$
$\frac{W + 88}{2.3} = h$

Therefore, we would write

$h(W) = \frac{W + 88}{2.3}$

c. $h(W(h)) = \frac{(50 + 2.3(h - 60)) + 88}{2.3}$
$= \frac{50 + 2.3h - 138 + 88}{2.3} = \frac{2.3h}{2.3}$
$= h$

$W(h(W)) = 50 + 2.3\left(\frac{W + 88}{2.3} - 60\right)$
$= 50 + W + 88 - 138$
$= W$

d. $h(80) = \frac{80 + 88}{2.3} = \frac{168}{2.3} \approx 73.04$

The height of a male who is at his ideal weight of 80 kg is roughly 73 inches.

90. a. $F = \frac{9}{5}C + 32$
$F - 32 = \frac{9}{5}C$
$\frac{5}{9}(F - 32) = C$

Therefore, we would write

$C(F) = \frac{5}{9}(F - 32)$

b. $C(F(C)) = \frac{5}{9}\left(\left(\frac{9}{5}C + 32\right) - 32\right)$
$= \frac{5}{9} \cdot \frac{9}{5}C = C$

$F(C(F)) = \frac{9}{5}\left(\frac{5}{9}(F - 32)\right) + 32$
$= F - 32 + 32 = F$

c. $C(70) = \frac{5}{9}(70 - 32) = \frac{5}{9}(38) \approx 21.1°C$

91. a. From the restriction given in the problem statement, the domain is $\{g \mid 29,700 \le g \le 71,950\}$ or $[29700, 71950]$.

b. $T(29700) = 4090 + 0.28(29700 - 29700)$
$= 4090$
$T(71950) = 4090 + 0.28(71950 - 29700)$
$= 15,920$

Since T is linear and increasing, we have that the range is $\{T \mid 4090 \le T \le 15,920\}$ or $[4090, 15920]$.

c. $T = 4090 + 0.28(g - 29,700)$
$T = 4090 + 0.28g - 8316$
$T = 0.28g - 4226$
$T + 4226 = 0.28g$
$\frac{T + 4226}{0.28} = g$

Therefore, we would write

$g(T) = \frac{T + 4,226}{0.28}$

Domain: $\{T \mid 4090 \le T \le 15,920\}$

Range: $\{g \mid 29,700 \le g \le 71,950\}$

92. a. From the restriction given in the problem statement, the domain is $\{g \mid 14,600 \le g \le 59,400\}$ or $[14600, 59400]$.

b. $T(14,600) = 1460 + 0.15(14,600 - 14,600)$
$= 1460$
$T(59,400) = 1460 + 0.15(59,400 - 14,600)$
$= 8180$
Since T is linear and increasing, we have that the range is $\{T \mid 1460 \le T \le 8180\}$ or $[1460, 8180]$.

c.

$$T = 1460 + 0.15(g - 14,600)$$
$$T = 1460 + 0.15g - 2190$$
$$T = 0.15g - 730$$
$$T + 730 = 0.15g$$
$$\frac{T+730}{0.15} = g$$

Therefore, we would write

$$g(T) = \frac{T+730}{0.15}$$

Domain: $\{T \mid 1460 \le T \le 8180\}$

Range: $\{g \mid 14,600 \le g \le 59,400\}$

93. a. The graph of H is symmetric about the y-axis. Since t represents the number of seconds *after* the rock begins to fall, we know that $t \ge 0$. The graph is strictly decreasing over its domain, so it is one-to-one.

b.

$$H = 100 - 4.9t^2$$
$$H + 4.9t^2 = 100$$
$$4.9t^2 = 100 - H$$
$$t^2 = \frac{100-H}{4.9}$$
$$t = \sqrt{\frac{100-H}{4.9}}$$

Therefore, we would write

$$t(H) = \sqrt{\frac{100-H}{4.9}}$$

(note: we only need the principal square root since we know $t \ge 0$)

$$H(t(H)) = 100 - 4.9\left(\sqrt{\frac{100-H}{4.9}}\right)^2$$
$$= 100 - 4.9\left(\frac{100-H}{4.9}\right)$$
$$= 100 - 100 + H$$
$$= H$$

$$t(H(t)) = \sqrt{\frac{100-(100-4.9t^2)}{4.9}}$$
$$= \sqrt{\frac{4.9t^2}{4.9}} = \sqrt{t^2} = t \quad (\text{since } t \ge 0)$$

c. $t(80) = \sqrt{\dfrac{100-80}{4.9}} \approx 2.02$

It will take the rock about 2.02 seconds to fall 80 meters.

94. a.

$$T(l) = 2\pi\sqrt{\frac{l}{32.2}}$$
$$T = 2\pi\sqrt{\frac{l}{32.2}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{32.2}}$$
$$\frac{T^2}{4\pi^2} = \frac{l}{32.2}$$
$$l = \frac{32.2T^2}{4\pi^2}$$
$$l(T) = \frac{8.05T^2}{\pi^2} = 8.05\left(\frac{T}{\pi}\right)^2, \quad T > 0$$

b. $l(3) = 8.05\left(\dfrac{3}{\pi}\right)^2 \approx 7.34$

A pendulum whose period is 3 seconds will be about 7.34 feet long.

95. $f(x)=\dfrac{ax+b}{cx+d}$

$$y=\frac{ax+b}{cx+d}$$
$$x=\frac{ay+b}{cy+d}\quad \text{Inverse}$$
$$x(cy+d)=ay+b$$
$$cxy+dx=ay+b$$
$$cxy-ay=b-dx$$
$$y(cx-a)=b-dx$$
$$y=\frac{b-dx}{cx-a}$$
$$f^{-1}(x)=\frac{-dx+b}{cx-a}$$

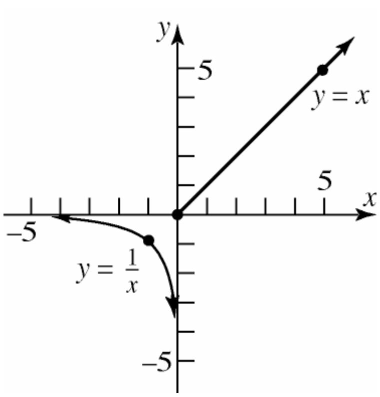
Now, $f=f^{-1}$ provided that $\dfrac{ax+b}{cx+d}=\dfrac{-dx+b}{cx-a}$.
This is only true if $a=-d$.

96. Yes. In order for a one-to-one function and its inverse to be equal, it graph must be symmetric about the line $y=x$. One such example is the function $f(x)=\dfrac{1}{x}$.

97. Answers will vary.

98. Answers will vary. One example is

$$f(x)=\begin{cases}\dfrac{1}{x}, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$



This function is one-to-one since the graph passes the Horizontal Line Test. However, the function is neither increasing nor decreasing on its domain.

99. The only way the function $y=f(x)$ can be both even and one-to-one is if the domain of $y=f(x)$ is $\{x \mid x\geq 0\}$. Otherwise, its graph will fail the Horizontal Line Test.

100. No, not every odd function is one-to-one. For example, $f(x)=x^3-x$ is an odd function, but it is not one-to-one.

101. If the graph of a function and its inverse intersect, they must intersect at a point on the line $y=x$. They cannot intersect anywhere else. The graphs do not have to intersect.

102. $C^{-1}(800,000)$ represents the number of cars manufactured for \$800,000.

Section 4.3

1. $4^3=64$; $8^{2/3}=\left(\sqrt[3]{8}\right)^2=2^2=4$; $3^{-2}=\dfrac{1}{3^2}=\dfrac{1}{9}$

2. $5x-2=3$
$$5x=5$$
$$x=1$$
The solution set is $\{1\}$.

3. False. To obtain the graph of $y=(x-2)^3$, we would shift the graph of $y=x^3$ to the *right* 2 units.

4.
$$\frac{f(x)-f(c)}{x-c}=\frac{3x-5-(3c-5)}{x-c}=\frac{3x-5-3c+5}{x-c}=\frac{3x-3c}{x-c}=\frac{3(x-c)}{x-c}=3$$

5. True

6. $(0,1)$, $(1,a)$, $\left(-1,\dfrac{1}{a}\right)$

7. 1

8. 4

9. False

10. False. The range will be $\{x \mid x > 0\}$ or $(0, \infty)$.

11. **a.** $3^{2.2} \approx 11.212$

b. $3^{2.23} \approx 11.587$

c. $3^{2.236} \approx 11.664$

d. $3^{\sqrt{5}} \approx 11.665$

12. **a.** $5^{1.7} \approx 15.426$

b. $5^{1.73} \approx 16.189$

c. $5^{1.732} \approx 16.241$

d. $5^{\sqrt{3}} \approx 16.242$

13. **a.** $2^{3.14} \approx 8.815$

b. $2^{3.141} \approx 8.821$

c. $2^{3.1415} \approx 8.824$

d. $2^{\pi} \approx 8.825$

14. **a.** $2^{2.7} \approx 6.498$

b. $2^{2.71} \approx 6.543$

c. $2^{2.718} \approx 6.580$

d. $2^{e} \approx 6.581$

15. **a.** $3.1^{2.7} \approx 21.217$

b. $3.14^{2.71} \approx 22.217$

c. $3.141^{2.718} \approx 22.440$

d. $\pi^{e} \approx 22.459$

16. **a.** $2.7^{3.1} \approx 21.738$

b. $2.71^{3.14} \approx 22.884$

c. $2.718^{3.141} \approx 23.119$

d. $e^{\pi} \approx 23.141$

17. $e^{1.2} \approx 3.320$

18. $e^{-1.3} \approx 0.273$

19. $e^{-0.85} \approx 0.427$

20. $e^{2.1} \approx 8.166$

21.

x	$y = f(x)$	$\frac{f(x+1)}{f(x)}$
-1	3	$\frac{6}{3} = 2$
0	6	$\frac{12}{6} = 2$
1	12	$\frac{18}{12} = \frac{3}{2}$
2	18	
3	30	

Not an exponential function since the ratio of consecutive terms is not constant.

22.

x	$y = g(x)$	$\frac{g(x+1)}{g(x)}$
-1	2	$\frac{5}{2}$
0	5	$\frac{8}{5}$
1	8	
2	11	
3	14	

Not an exponential function since the ratio of consecutive terms is not constant.

23.

x	$y = H(x)$	$\frac{H(x+1)}{H(x)}$
-1	$\frac{1}{4}$	$\frac{1}{(1/4)} = 4$
0	1	$\frac{4}{1} = 4$
1	4	$\frac{16}{4} = 4$
2	16	$\frac{64}{16} = 4$
3	64	

Yes, an exponential function since the ratio of consecutive terms is constant with $a = 4$. So the base is 4.

24.

x	$y=F(x)$	$\frac{F(x+1)}{F(x)}$
–1	$\frac{2}{3}$	$\frac{1}{(2/3)}=1\cdot\frac{3}{2}=\frac{3}{2}$
0	1	$\frac{(3/2)}{1}=\frac{3}{2}$
1	$\frac{3}{2}$	$\frac{(9/4)}{(3/2)}=\frac{9}{4}\cdot\frac{2}{3}=\frac{3}{2}$
2	$\frac{9}{4}$	$\frac{(27/8)}{(9/4)}=\frac{27}{8}\cdot\frac{4}{9}=\frac{3}{2}$
3	$\frac{27}{8}$	

Yes, an exponential function since the ratio of consecutive terms is constant with $a=\frac{3}{2}$. So the base is $\frac{3}{2}$.

25.

x	$y=f(x)$	$\frac{f(x+1)}{f(x)}$
–1	$\frac{3}{2}$	$\frac{3}{(3/2)}=3\cdot\frac{2}{3}=2$
0	3	$\frac{6}{3}=2$
1	6	$\frac{12}{6}=2$
2	12	$\frac{24}{12}=2$
3	24	

Yes, an exponential function since the ratio of consecutive terms is constant with $a=2$. So the base is 2.

26.

x	$y=g(x)$	$\frac{g(x+1)}{g(x)}$
–1	6	$\frac{1}{6}$
0	1	$\frac{0}{1}=0$
1	0	
2	3	
3	10	

Not an exponential function since the ratio of consecutive terms is not constant.

27.

x	$y=H(x)$	$\frac{H(x+1)}{H(x)}$
–1	2	$\frac{4}{2}=2$
0	4	$\frac{6}{4}=\frac{3}{2}$
1	6	
2	8	
3	10	

Not an exponential function since the ratio of consecutive terms is not constant.

28.

x	$y=f(x)$	$\frac{f(x+1)}{f(x)}$
–1	$\frac{1}{2}$	$\frac{(1/4)}{(1/2)}=\frac{1}{4}\cdot\frac{2}{1}=\frac{1}{2}$
0	$\frac{1}{4}$	$\frac{(1/8)}{(1/4)}=\frac{1}{8}\cdot\frac{4}{1}=\frac{1}{2}$
1	$\frac{1}{8}$	$\frac{(1/16)}{(1/8)}=\frac{1}{16}\cdot\frac{8}{1}=\frac{1}{2}$
2	$\frac{1}{16}$	$\frac{(1/32)}{(1/16)}=\frac{1}{32}\cdot\frac{16}{1}=\frac{1}{2}$
3	$\frac{1}{32}$	

Yes, an exponential function since the ratio of consecutive terms is constant with $a=\frac{1}{2}$. So The base is $\frac{1}{2}$.

29. *B*

30. *F*

31. *D*

32. *H*

33. *A*

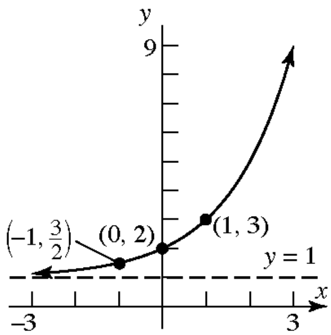
34. *C*

35. *E*

36. *G*

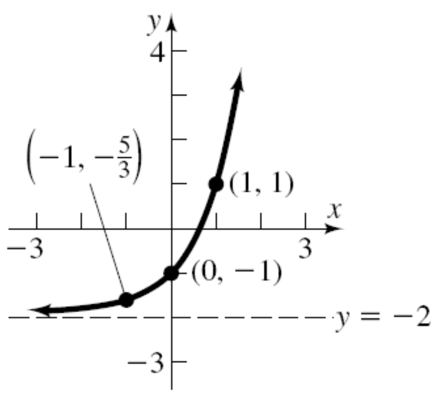
37. $f(x) = 2^x + 1$

Using the graph of $y = 2^x$, shift the graph up 1 unit.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(1, \infty)$
Horizontal Asymptote: $y = 1$



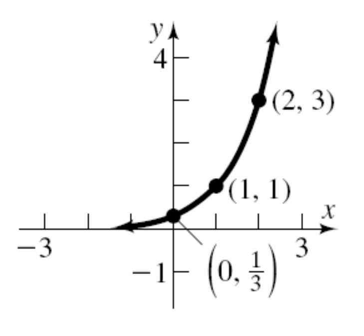
38. $f(x) = 3^x - 2$

Using the graph of $y = 3^x$, shift the graph down 2 units.
Domain: All real numbers
Range: $\{y \mid y > -2\}$ or $(-2, \infty)$
Horizontal Asymptote: $y = -2$



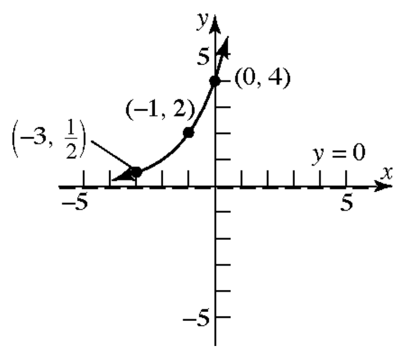
39. $f(x) = 3^{x-1}$

Using the graph of $y = 3^x$, shift the graph right 1 unit.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



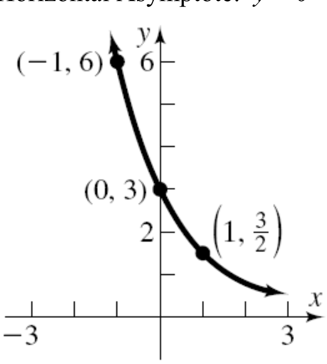
40. $f(x) = 2^{x+2}$

Using the graph of $y = 2^x$, shift the graph left 2 units.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



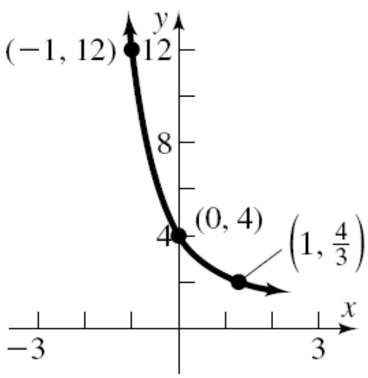
41. $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x$

Using the graph of $y = \left(\frac{1}{2}\right)^x$, vertically stretch the graph by a factor of 3. That is, for each point on the graph, multiply the y-coordinate by 3.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



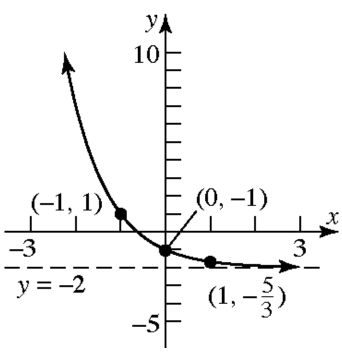
42. $f(x)=4\left(\frac{1}{3}\right)^x$

Using the graph of $y=\left(\frac{1}{3}\right)^x$, vertically stretch the graph by a factor of 4. That is, for each point on the graph, multiply the y-coordinate by 4.
Domain: All real numbers
Range: $\{y\mid y>0\}$ or $(0,\infty)$
Horizontal Asymptote: $y=0$



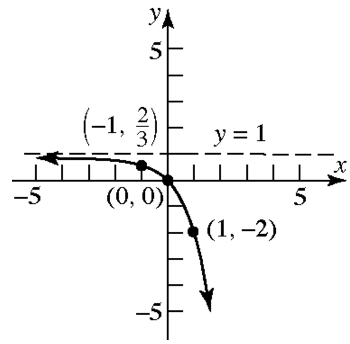
43. $f(x)=3^{-x}-2$

Using the graph of $y=3^x$, reflect the graph about the y-axis, and shift down 2 units.
Domain: All real numbers
Range: $\{y\mid y>-2\}$ or $(-2,\infty)$
Horizontal Asymptote: $y=-2$



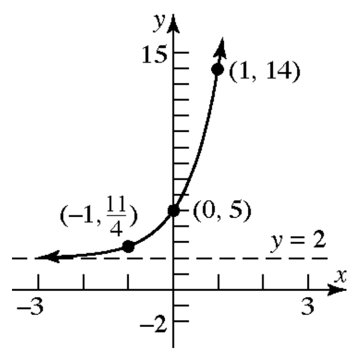
44. $f(x)=-3^x+1$

Using the graph of $y=3^x$, reflect the graph about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y\mid y<1\}$ or $(-\infty,1)$
Horizontal Asymptote: $y=1$



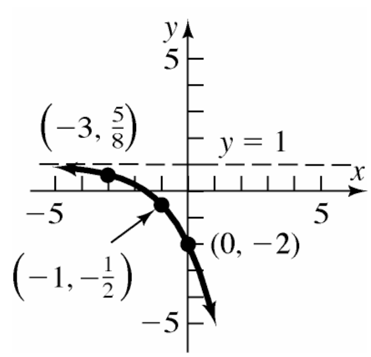
45. $f(x)=2+3\left(4^x\right)$

Using the graph of $y=4^x$, stretch the graph vertically by a factor of 3, and shift up 2 units.
Domain: All real numbers
Range: $\{y\mid y>2\}$ or $(2,\infty)$
Horizontal Asymptote: $y=2$



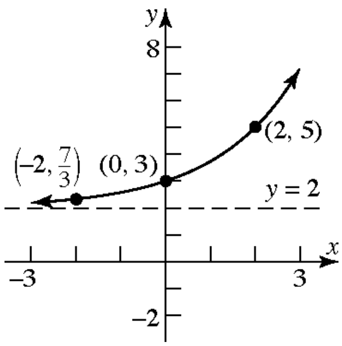
46. $f(x)=1-3\left(2^x\right)$

Using the graph of $y=2^x$, stretch the graph vertically by a factor of 3, reflect about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y\mid y<1\}$ or $(-\infty,1)$
Horizontal Asymptote: $y=1$



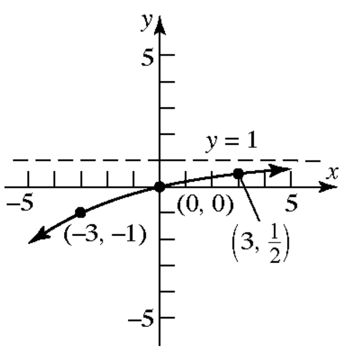
47. $f(x) = 2 + 3^{x/2}$

Using the graph of $y = 3^x$, stretch the graph horizontally by a factor of 2, and shift up 2 units.
Domain: All real number
Range: $\{y \mid y > 2\}$ or $(2, \infty)$
Horizontal Asymptote: $y = 2$



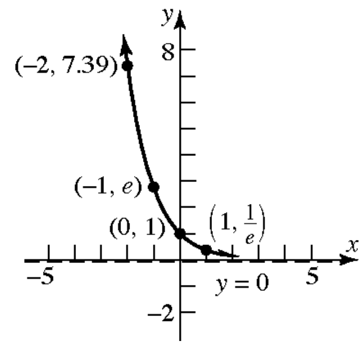
48. $f(x) = 1 - 2^{-x/3}$

Using the graph of $y = 2^x$, stretch the graph horizontally by a factor of 3, reflect about the y-axis, reflect about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y \mid y < 1\}$ or $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



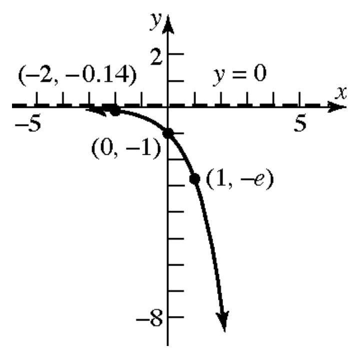
49. $f(x) = e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



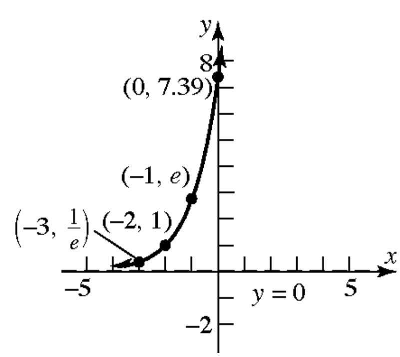
50. $f(x) = -e^x$

Using the graph of $y = e^x$, reflect the graph about the x-axis.
Domain: All real numbers
Range: $\{y \mid y < 0\}$ or $(-\infty, 0)$
Horizontal Asymptote: $y = 0$



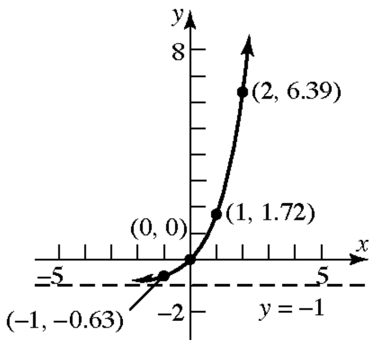
51. $f(x) = e^{x+2}$

Using the graph of $y = e^x$, shift the graph 2 units to the left.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



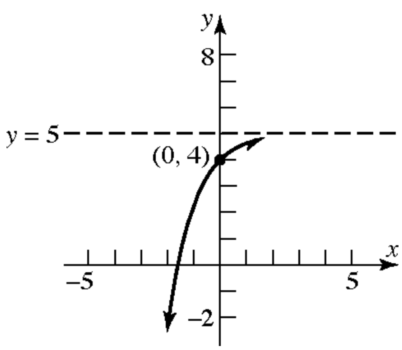
52. $f(x) = e^x - 1$

Using the graph of $y = e^x$, shift the graph down 1 unit.
Domain: All real numbers
Range: $\{y \mid y > -1\}$ or $(-1, \infty)$
Horizontal Asymptote: $y = -1$



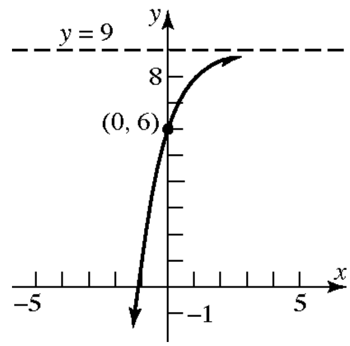
53. $f(x) = 5 - e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 5 units.
Domain: All real numbers
Range: $\{y \mid y < 5\}$ or $(-\infty, 5)$
Horizontal Asymptote: $y = 5$



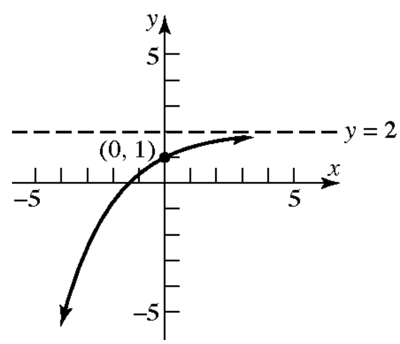
54. $f(x) = 9 - 3e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 9 units.
Domain: All real numbers
Range: $\{y \mid y < 9\}$ or $(-\infty, 9)$
Horizontal Asymptote: $y = 9$



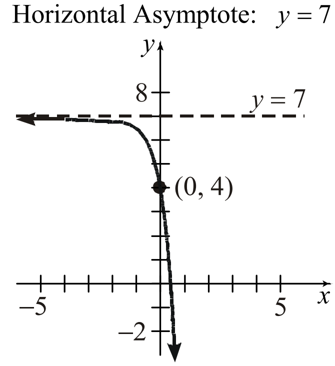
55. $f(x) = 2 - e^{-x/2}$

Using the graph of $y = e^x$, reflect the graph about the y-axis, stretch horizontally by a factor of 2, reflect about the x-axis, and shift up 2 units.
Domain: All real numbers
Range: $\{y \mid y < 2\}$ or $(-\infty, 2)$
Horizontal Asymptote: $y = 2$



56. $f(x) = 7 - 3e^{-2x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis, shrink horizontally by a factor of $\frac{1}{2}$, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 7 units.
Domain: All real numbers
Range: $\{y \mid y < 7\}$ or $(-\infty, 7)$
Horizontal Asymptote: $y = 7$



57. $7^x = 7^3$

We have a single term with the same base on both sides of the equation. Therefore, we can set the exponents equal to each other: $x = 3$.

The solution set is $\{3\}$.

58. $5^x = 5^{-6}$

We have a single term with the same base on both sides of the equation. Therefore, we can set the exponents equal to each other: $x = -6$.

The solution set is $\{-6\}$.

59. $2^{-x} = 16$

$$2^{-x} = 2^4$$
$$-x = 4$$
$$x = -4$$

The solution set is $\{-4\}$.

60. $3^{-x} = 81$

$$3^{-x} = 3^4$$
$$-x = 4$$
$$x = -4$$

The solution set is $\{-4\}$.

61. $\left(\frac{1}{5}\right)^x = \frac{1}{25}$

$$\left(\frac{1}{5}\right)^x = \frac{1}{5^2}$$
$$\left(\frac{1}{5}\right)^x = \left(\frac{1}{5}\right)^2$$
$$x = 2$$

The solution set is $\{2\}$.

62. $\left(\frac{1}{4}\right)^x = \frac{1}{64}$

$$\left(\frac{1}{4}\right)^x = \frac{1}{4^3}$$
$$\left(\frac{1}{4}\right)^x = \left(\frac{1}{4}\right)^3$$
$$x = 3$$

The solution set is $\{3\}$.

63. $2^{2x-1} = 4$

$$2^{2x-1} = 2^2$$
$$2x - 1 = 2$$
$$2x = 3$$
$$x = \frac{3}{2}$$

The solution set is $\left\{\frac{3}{2}\right\}$.

64. $5^{x+3} = \frac{1}{5}$

$$5^{x+3} = 5^{-1}$$
$$x + 3 = -1$$
$$x = -4$$

The solution set is $\{-4\}$.

65. $3^{x^3} = 9^x$

$$3^{x^3} = \left(3^2\right)^x$$
$$3^{x^3} = 3^{2x}$$
$$x^3 = 2x$$
$$x^3 - 2x = 0$$
$$x\left(x^2 - 2\right) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

The solution set is $\left\{-\sqrt{2}, 0, \sqrt{2}\right\}$.

66. $4^{x^2} = 2^x$

$$\left(2^2\right)^{x^2} = 2^x$$
$$2^{2x^2} = 2^x$$
$$2x^2 = x$$
$$2x^2 - x = 0$$
$$x(2x - 1) = 0$$
$$x = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$2x = 1$$
$$x = \frac{1}{2}$$

The solution set is $\left\{0, \frac{1}{2}\right\}$.

67.
$$\begin{aligned} 8^{-x+14} &= 16^{x} \\ \left(2^{3}\right)^{-x+14} &= \left(2^{4}\right)^{x} \\ 2^{-3x+42} &= 2^{4x} \\ -3x+42 &= 4x \\ 42 &= 7x \\ 6 &= x \end{aligned}$$
The solution set is $\{6\}$.

68.
$$\begin{aligned} 9^{-x+15} &= 27^{x} \\ \left(3^{2}\right)^{-x+15} &= \left(3^{3}\right)^{x} \\ 3^{-2x+30} &= 3^{3x} \\ -2x+30 &= 3x \\ 30 &= 5x \\ 6 &= x \end{aligned}$$
The solution set is $\{6\}$.

69.
$$\begin{aligned} 3^{x^2-7} &= 27^{2x} \\ 3^{x^2-7} &= \left(3^{3}\right)^{2x} \\ 3^{x^2-7} &= 3^{6x} \\ x^2-7 &= 6x \\ x^2-6x-7 &= 0 \\ (x-7)(x+1) &= 0 \end{aligned}$$
$x-7=0$ or $x+1=0$
$x=7$ $\quad x=-1$
The solution set is $\{-1,7\}$.

70.
$$\begin{aligned} 5^{x^2+8} &= 125^{2x} \\ 5^{x^2+8} &= \left(5^{3}\right)^{2x} \\ 5^{x^2+8} &= 5^{6x} \\ x^2+8 &= 6x \\ x^2-6x+8 &= 0 \\ (x-4)(x-2) &= 0 \end{aligned}$$
$x-4=0$ or $x-2=0$
$x=4$ $\quad x=2$
The solution set is $\{2,4\}$.

71.
$$\begin{aligned} 4^{x}\cdot 2^{x^2} &= 16^{2} \\ \left(2^{2}\right)^{x}\cdot 2^{x^2} &= \left(2^{4}\right)^{2} \\ 2^{2x}\cdot 2^{x^2} &= 2^{8} \\ 2^{2x+x^2} &= 2^{8} \\ x^2+2x &= 8 \\ x^2+2x-8 &= 0 \\ (x+4)(x-2) &= 0 \end{aligned}$$
$x+4=0$ or $x-2=0$
$x=-4$ $\quad x=2$
The solution set is $\{-4,2\}$.

72.
$$\begin{aligned} 9^{2x}\cdot 27^{x^2} &= 3^{-1} \\ \left(3^{2}\right)^{2x}\cdot\left(3^{3}\right)^{x^2} &= 3^{-1} \\ 3^{4x}\cdot 3^{3x^2} &= 3^{-1} \\ 3^{4x+3x^2} &= 3^{-1} \\ 3x^2+4x &= -1 \\ 3x^2+4x+1 &= 0 \\ (3x+1)(x+1) &= 0 \end{aligned}$$
$3x+1=0$ or $x+1=0$
$3x=-1$ $\quad x=-1$
$x=-\frac{1}{3}$
The solution set is $\left\{-1,-\frac{1}{3}\right\}$.

73.
$$\begin{aligned} e^{x} &= e^{3x+8} \\ x &= 3x+8 \\ -2x &= 8 \\ x &= -4 \end{aligned}$$
The solution set is $\{-4\}$.

74.
$$\begin{aligned} e^{3x} &= e^{2-x} \\ 3x &= 2-x \\ 4x &= 2 \\ x &= \frac{1}{2} \end{aligned}$$
The solution set is $\left\{\frac{1}{2}\right\}$.

75.
$$e^{x^2}=e^{3x}\cdot\frac{1}{e^2}$$
$$e^{x^2}=e^{3x}\cdot e^{-2}$$
$$e^{x^2}=e^{3x-2}$$
$$x^2=3x-2$$
$$x^2-3x+2=0$$
$$(x-2)(x-1)=0$$
$$x-2=0 \quad\text{or}\quad x-1=0$$
$$x=2 \qquad\qquad x=1$$
The solution set is $\{1,2\}$.

76.
$$\left(e^4\right)^x\cdot e^{x^2}=e^{12}$$
$$e^{4x}\cdot e^{x^2}=e^{12}$$
$$e^{4x+x^2}=e^{12}$$
$$x^2+4x=12$$
$$x^2+4x-12=0$$
$$(x+6)(x-2)=0$$
$$x+6=0 \quad\text{or}\quad x-2=0$$
$$x=-6 \qquad\qquad x=2$$
The solution set is $\{-6,2\}$.

77. a. $f(4)=2^4=16$

The point $(4,16)$ is on the graph of *f*.

b.
$$f(x)=\frac{1}{16}$$
$$2^x=\frac{1}{16}$$
$$2^x=\frac{1}{2^4}$$
$$2^x=2^{-4}$$
$$x=-4$$
The point $\left(-4,\frac{1}{16}\right)$ is on the graph of *f*.

78. a. $f(4)=3^4=81$

The point $(4,81)$ is on the graph of *f*.

b.
$$f(x)=\frac{1}{9}$$
$$3^x=\frac{1}{9}$$
$$3^x=\frac{1}{3^2}$$
$$3^x=3^{-2}$$
$$x=-2$$
The point $\left(-2,\frac{1}{9}\right)$ is on the graph of *f*.

79. a. $g(-1)=4^{-1}+2=\frac{1}{4}+2=\frac{9}{4}$

The point $\left(-1,\frac{9}{4}\right)$ is on the graph of *g*.

b.
$$g(x)=66$$
$$4^x+2=66$$
$$4^x=64$$
$$4^x=4^3$$
$$x=3$$
The point $(3,66)$ is on the graph of *g*.

80. a. $g(-1)=5^{-1}-3=\frac{1}{5}-3=-\frac{14}{5}$

The point $\left(-1,-\frac{14}{15}\right)$ is on the graph of *g*.

b.
$$g(x)=122$$
$$5^x-3=122$$
$$5^x=125$$
$$5^x=5^3$$
$$x=3$$
The point $(3,122)$ is on the graph of *g*.

81. a. $H(-2)=3\left(\frac{1}{2}\right)^{-2}-2=3(2)^2-2=10$

The point $(-2,10)$ is on the graph of *H*.

b. $$H(x) = -\frac{13}{8}$$
$$3\left(\frac{1}{2}\right)^x - 2 = -\frac{13}{8}$$
$$3\left(\frac{1}{2}\right)^x = \frac{3}{8}$$
$$\left(\frac{1}{2}\right)^x = \frac{1}{8}$$
$$\left(\frac{1}{2}\right)^x = \frac{1}{2^3}$$
$$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^3$$
$$x = 3$$

The point $\left(3, -\frac{13}{8}\right)$ is on the graph of *H*.

82. a. $F(-1) = -2\left(\frac{1}{3}\right)^{-1} + 1 = -2(3)^1 + 1 = -5$

The point $(-1, -5)$ is on the graph of *F*.

b. $$F(x) = -53$$
$$-2\left(\frac{1}{3}\right)^x + 1 = -53$$
$$-2\left(\frac{1}{3}\right)^x = -54$$
$$\left(\frac{1}{3}\right)^x = 27$$
$$3^{-x} = 3^3$$
$$-x = 3$$
$$x = -3$$

The point $(-3, -53)$ is on the graph of *F*.

83. If $4^x = 7$, then $\left(4^x\right)^{-2} = 7^{-2}$
$$4^{-2x} = \frac{1}{7^2}$$
$$4^{-2x} = \frac{1}{49}$$

84. If $2^x = 3$, then $\left(2^x\right)^{-2} = 3^{-2}$
$$2^{-2x} = \frac{1}{3^2}$$
$$\left(2^2\right)^{-x} = \frac{1}{9}$$
$$4^{-x} = \frac{1}{9}$$

85. If $3^{-x} = 2$, then $\left(3^{-x}\right)^{-2} = 2^{-2}$
$$3^{2x} = \frac{1}{2^2}$$
$$3^{2x} = \frac{1}{4}$$

86. If $5^{-x} = 3$, then $\left(5^{-x}\right)^{-3} = 3^{-3}$
$$5^{3x} = \frac{1}{3^3}$$
$$5^{3x} = \frac{1}{27}$$

87. We need a function of the form $f(x) = k \cdot a^{p \cdot x}$, with $a > 0,\ a \neq 1$. The graph contains the points $\left(-1, \frac{1}{3}\right)$, $(0,1)$, $(1,3)$, and $(2,9)$. In other words, $f(-1) = \frac{1}{3}$, $f(0) = 1$, $f(1)=3$, and $f(2) = 9$.

Therefore, $f(0) = k \cdot a^{p \cdot (0)}$
$$1 = k \cdot a^0$$
$$1 = k \cdot 1$$
$$1 = k$$

and $f(1) = a^{p \cdot (1)}$
$$3 = a^p$$

Let's use $a = 3,\ p = 1$. Then $f(x) = 3^x$. Now we need to verify that this function yields the other known points on the graph. $f(-1) = 3^{-1} = \frac{1}{3}$;

$f(2) = 3^2 = 9$

So we have the function $f(x) = 3^x$.

88. We need a function of the form $f(x)=k\cdot a^{p\cdot x}$, with $a>0,\ a\neq 1$. The graph contains the points $\left(-1,\frac{1}{5}\right)$, $(0,1)$, and $(1,5)$. In other words, $f(-1)=\frac{1}{5}$, $f(0)=1$, and $f(1)=5$. Therefore,

$$\begin{aligned} f(0)&=k\cdot a^{p\cdot(0)}\\ 1&=k\cdot a^0\\ 1&=k\cdot 1\\ 1&=k \end{aligned}$$

and $f(1)=a^{p\cdot(1)}$

$$5=a^p$$

Let's use $a=5,\ p=1$. Then $f(x)=5^x$. Now we need to verify that this function yields the other known point on the graph.

$$f(-1)=5^{-1}=\frac{1}{5}$$

So we have the function $f(x)=5^x$.

89. We need a function of the form $f(x)=k\cdot a^{p\cdot x}$, with $a>0,\ a\neq 1$. The graph contains the points $\left(-1,-\frac{1}{6}\right)$, $(0,-1)$, $(1,-6)$, and $(2,-36)$. In other words, $f(-1)=-\frac{1}{6}$, $f(0)=-1$, $f(1)=-6$, and $f(2)=-36$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$

$$\begin{aligned} -1&=k\cdot a^0\\ -1&=k\cdot 1\\ -1&=k \end{aligned}$$

and $f(1)=-a^{p\cdot(1)}$

$$\begin{aligned} -6&=-a^p\\ 6&=a^p \end{aligned}$$

Let's use $a=6,\ p=1$. Then $f(x)=-6^x$.

Now we need to verify that this function yields the other known points on the graph.

$$f(-1)=-6^{-1}=-\frac{1}{6}$$

$$f(2)=-6^2=-36$$

So we have the function $f(x)=-6^x$.

90. We need a function of the form $f(x)=k\cdot a^{p\cdot x}$, with $a>0,\ a\neq 1$. The graph contains the points $\left(-1,-\frac{1}{e}\right)$, $(0,-1)$, $(1,-e)$, and $\left(2,-e^2\right)$. In other words, $f(-1)=-\frac{1}{e}$, $f(0)=-1$, $f(1)=-e$, and $f(2)=-e^2$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$

$$\begin{aligned} -1&=k\cdot a^0\\ -1&=k\cdot 1\\ -1&=k \end{aligned}$$

and $f(1)=-a^{p\cdot(1)}$

$$\begin{aligned} -e&=-a^p\\ e&=a^p \end{aligned}$$

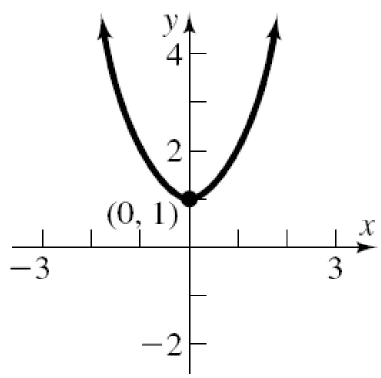
Let's use $a=e,\ p=1$. Then $f(x)=-e^x$. Now we need to verify that this function yields the other known points on the graph.

$$f(-1)=-e^{-1}=-\frac{1}{e}$$

$$f(2)=-e^2$$

So we have the function $f(x)=-e^x$.

91. $f(x)=\begin{cases} e^{-x} & \text{if } x<0\\ e^{x} & \text{if } x\geq 0\end{cases}$

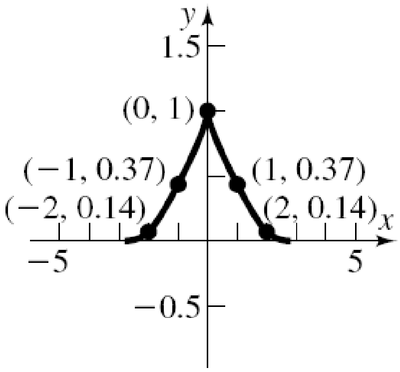


Domain: $(-\infty,\infty)$

Range: $\{y\mid y\geq 1\}$ or $[1,\infty)$

Intercept: $(0,1)$

92. $f(x)=\begin{cases} e^{x} & \text{if } x<0 \\ e^{-x} & \text{if } x\ge 0 \end{cases}$

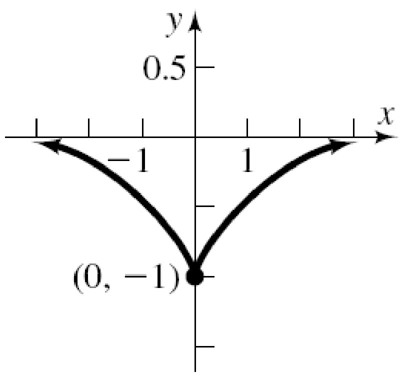


Domain: $(-\infty,\infty)$

Range: $\{y\,|\,0<y\le 1\}$ or $(0,1]$

Intercept: $(0,1)$

93. $f(x)=\begin{cases} -e^{x} & \text{if } x<0 \\ -e^{-x} & \text{if } x\ge 0 \end{cases}$

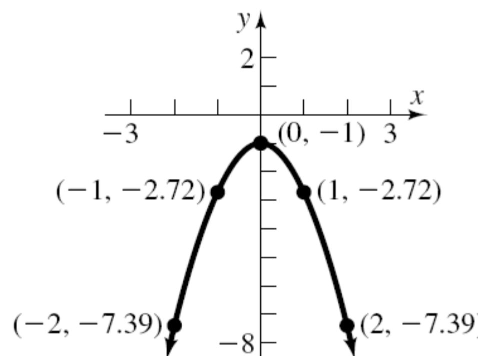


Domain: $(-\infty,\infty)$

Range: $\{y\,|\,-1\le y<0\}$ or $[-1,0)$

Intercept: $(0,-1)$

94. $f(x)=\begin{cases} -e^{-x} & \text{if } x<0 \\ -e^{x} & \text{if } x\ge 0 \end{cases}$



Domain: $(-\infty,\infty)$

Range: $\{y\,|\,y\le -1\}$ or $(-\infty,-1]$

Intercept: $(0,-1)$

95. $p(n)=100(0.97)^{n}$

a. $p(10)=100(0.97)^{10}\approx 74\%$ of light

b. $p(25)=100(0.97)^{25}\approx 47\%$ of light

96. $p(h)=760e^{-0.145h}$

a. $$\begin{aligned} p(2) &= 760e^{-0.145(2)} \\ &= 760e^{-0.290} \\ &\approx 568.68 \text{ mm of mercury} \end{aligned}$$

b. $$\begin{aligned} p(10) &= 760e^{-0.145(10)} \\ &= 760e^{-1.45} \\ &\approx 178.27 \text{ mm of mercury} \end{aligned}$$

97. $p(x)=16,630(0.90)^{x}$

a. $p(3)=16,630(0.90)^{3}\approx \$12,123$

b. $p(9)=16,630(0.90)^{9}\approx \$6,443$

98. $A(n)=A_0e^{-0.35n}$

a. $$\begin{aligned} A(3) &= 100e^{-0.35(3)} \\ &= 100e^{-1.05} \\ &\approx 34.99 \text{ square millimeters} \end{aligned}$$

b. $$\begin{aligned} A(10) &= 100e^{-0.35(10)} \\ &= 100e^{-3.5} \\ &\approx 3.02 \text{ square millimeters} \end{aligned}$$

99. $D(h)=5e^{-0.4h}$

$D(1)=5e^{-0.4(1)}=5e^{-0.4}\approx 3.35$

After 1 hours, 3.35 milligrams will be present.

$D(6)=5e^{-0.4(6)}=5e^{-2.4}\approx 0.45$ milligrams

After 6 hours, 0.45 milligrams will be present.

100. $N=P\left(1-e^{-0.15d}\right)$

$$\begin{aligned}N(3)&=1000\left(1-e^{-0.15(3)}\right)\\&=1000\left(1-e^{-0.45}\right)\\&\approx 362\end{aligned}$$

After 3 days, 362 students will have heard the rumor.

101. $F(t)=1-e^{-0.1t}$

a. $F(10)=1-e^{-0.1(10)}=1-e^{-1}\approx 0.63$

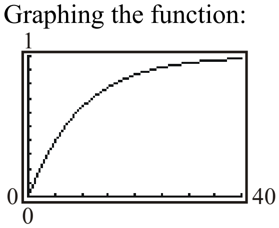
The probability that a car will arrive within 10 minutes of 12:00 PM is 0.63.

b. $F(40)=1-e^{-0.1(40)}=1-e^{-4}\approx 0.98$

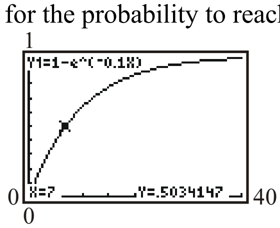
The probability that a car will arrive within 40 minutes of 12:00 PM is 0.98.

c. As $t\to\infty$, $F(t)=1-e^{-0.1t}\to 1-0=1$

d. Graphing the function:



e. $F(7)\approx 0.50$, so about 7 minutes are needed for the probability to reach 50%.



102. $F(t)=1-e^{-0.15t}$

a. $F(15)=1-e^{-0.15(15)}=1-e^{-2.25}\approx 0.895$

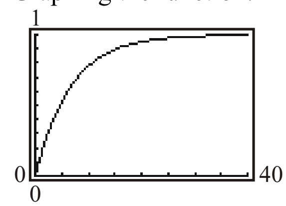
The probability that a car will arrive within 15 minutes of 5:00 PM is 0.895.

b. $F(30)=1-e^{-0.15(30)}=1-e^{-4.5}\approx 0.989$

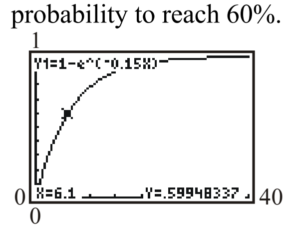
The probability that a car will arrive within 30 minutes of 5:00 PM is 0.989.

c. As $t\to\infty$, $F(t)=1-e^{-0.15t}\to 1-0=1$

d. Graphing the function:



e. $F(6)\approx 0.60$, so 6 minutes are needed for the probability to reach 60%.



103. $P(x)=\dfrac{20^x e^{-20}}{x!}$

a. $P(15)=\dfrac{20^{15}e^{-20}}{15!}\approx 0.0516$ or 5.16%

The probability that 15 cars will arrive between 5:00 PM and 6:00 PM is 5.16%.

b. $P(20)=\dfrac{20^{20}e^{-20}}{20!}\approx 0.0888$ or 8.88%

The probability that 20 cars will arrive between 5:00 PM and 6:00 PM is 8.88%.

104. $P(x)=\dfrac{4^x e^{-4}}{x!}$

a. $P(5)=\dfrac{4^5 e^{-4}}{5!}\approx 0.156$ or 15.6%

The probability that 5 people will arrive within the next minute is 15.6%.

b. $P(8)=\dfrac{4^8 e^{-4}}{8!}\approx 0.030$ or 3.0%

The probability that 8 people will arrive within the next minute is 3.0%.

105. $R=10^{\left(\frac{4221}{T+459.4}-\frac{4221}{D+459.4}+2\right)}$

a. $R=10^{\left(\frac{4221}{50+459.4}-\frac{4221}{41+459.4}+2\right)}\approx 70.95\%$

b. $R = 10^{\left(\frac{4221}{68+459.4} - \frac{4221}{59+459.4} + 2\right)} \approx 72.62\%$

c. $R = 10^{\left(\frac{4221}{T+459.4} - \frac{4221}{T+459.4} + 2\right)} = 10^2 = 100\%$

106. $L(t) = 500\left(1 - e^{-0.0061\,t}\right)$

a. $$\begin{aligned} L(30) &= 500\left(1 - e^{-0.0061(30)}\right) \\ &= 500\left(1 - e^{-0.183}\right) \\ &\approx 84 \end{aligned}$$

The student will learn about 84 words after 30 minutes.

b. $$\begin{aligned} L(60) &= 500\left(1 - e^{-0.0061(60)}\right) \\ &= 500\left(1 - e^{-0.366}\right) \\ &\approx 153 \end{aligned}$$

The student will learn about 153 words after 60 minutes.

107. $I = \frac{E}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$

a. $I_1 = \frac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)0.3}\right] = 12\left[1 - e^{-0.6}\right] \approx 5.414$

amperes after 0.3 second

$I_1 = \frac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)0.5}\right] = 12\left[1 - e^{-1}\right] \approx 7.585$

amperes after 0.5 second

$I_1 = \frac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)1}\right] = 12\left[1 - e^{-2}\right] \approx 10.376$

amperes after 1 second

b. As $t \to \infty$, $e^{-\left(\frac{10}{5}\right)t} \to 0$. Therefore, as,

$t \to \infty,\ I_1 = \frac{120}{10}\left[1 - e^{-\left(\frac{10}{5}\right)t}\right] \to 12[1-0] = 12$,

which means the maximum current is 12 amperes.

c. See the graph at the bottom of the page.

d. $$\begin{aligned} I_2 &= \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)0.3}\right] \\ &= 24\left[1 - e^{-0.15}\right] \\ &\approx 3.343 \text{ amperes after 0.3 second} \end{aligned}$$

$$\begin{aligned} I_2 &= \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)0.5}\right] \\ &= 24\left[1 - e^{-0.25}\right] \\ &\approx 5.309 \text{ amperes after 0.5 second} \end{aligned}$$

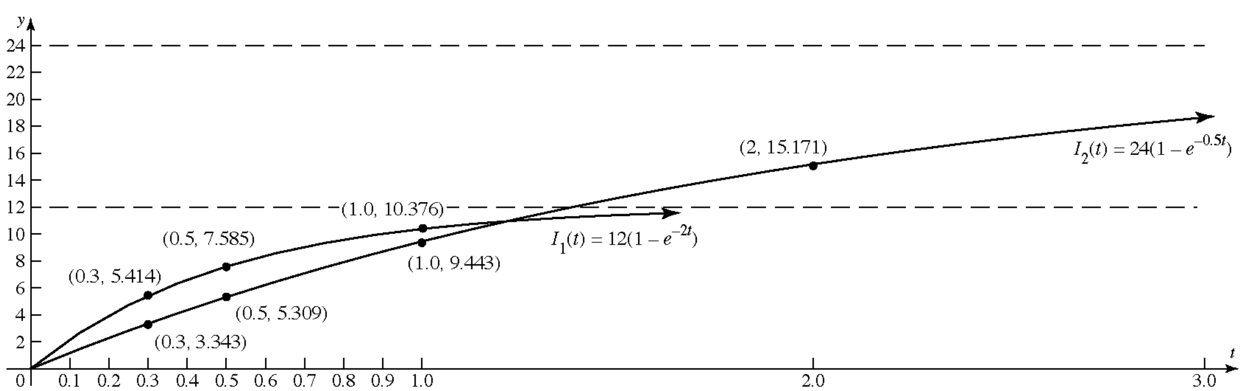
$$\begin{aligned} I_2 &= \frac{120}{5}\left[1 - e^{-\left(\frac{5}{10}\right)1}\right] \\ &= 24\left[1 - e^{-0.5}\right] \\ &\approx 9.443 \text{ amperes after 1 second} \end{aligned}$$

e. As $t \to \infty$, $e^{-\left(\frac{5}{10}\right)t} \to 0$. Therefore, as,

$t \to \infty,\ I_1 = \frac{120}{5}\left[1 - e^{-\left(\frac{10}{5}\right)t}\right] \to 24[1-0] = 24$,

which means the maximum current is 24 amperes.

f. See the graph at the bottom of the page.



108. $I=\frac{E}{R}\cdot e^{\left(\frac{-t}{RC}\right)}$

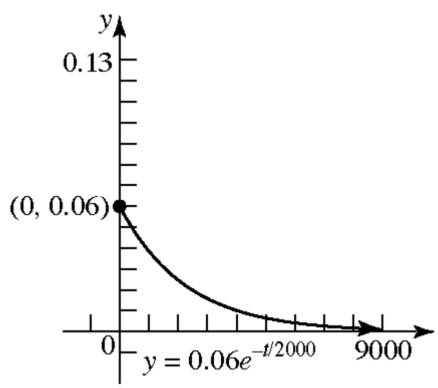
a. $I_1=\frac{120}{2000}\cdot e^{\left(\frac{-0}{2000\cdot 1}\right)}=\frac{120}{2000}e^{0}=0.06$ amperes initially.

$I_1=\frac{120}{2000}\cdot e^{\left(\frac{-1000}{2000\cdot 1}\right)}=\frac{120}{2000}e^{-1/2}\approx 0.0364$ amperes after 1000 microseconds

$I_1=\frac{120}{2000}\cdot e^{\left(\frac{-3000}{2000\cdot 1}\right)}=\frac{120}{2000}e^{-1.5}\approx 0.0134$ amperes after 3000 microseconds

b. The maximum current occurs at $t=0$. Therefore, the maximum current is 0.06 amperes.

c. Graphing the function:



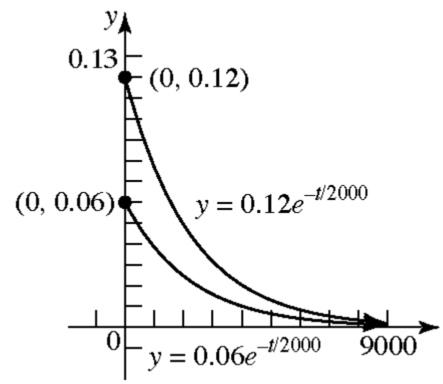
d. $I_2=\frac{120}{1000}\cdot e^{\left(\frac{-0}{1000\cdot 2}\right)}=\frac{120}{1000}e^{0}=0.12$ amperes initially.

$I_2=\frac{120}{1000}\cdot e^{\left(\frac{-1000}{1000\cdot 2}\right)}=\frac{120}{1000}e^{-1/2}\approx 0.0728$ amperes after 1000 microseconds

$I_2=\frac{120}{1000}\cdot e^{\left(\frac{-3000}{1000\cdot 2}\right)}=\frac{120}{1000}e^{-1.5}\approx 0.0268$ amperes after 3000 microseconds

e. The maximum current occurs at $t=0$. Therefore, the maximum current is 0.12 amperes.

f. Graphing the functions:



109. $2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots+\frac{1}{n!}$

$n=4;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}\approx 2.7083$

$n=6;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}\approx 2.7181$

$n=8;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}$
≈ 2.7182788

$n=10;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}+\frac{1}{9!}+\frac{1}{10!}$
≈ 2.7182818

$e\approx 2.718281828$

110. $n=1;\quad 2+\frac{1}{1}=3$

$n=2;\quad 2+\cfrac{1}{1+\cfrac{1}{1+1}}\approx 2.666666667$

$n=3;\quad 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3}}}\approx 2.727272727$

$n=4;\quad 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4}}}}\approx 2.716981132$

$n=5;\quad 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5}}}}}\approx 2.718446602$

$$n=6;\ 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6}}}}}}\approx 2.718263332$$

$$n=7;\ 2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+\cfrac{6}{7}}}}}}}\approx 2.718283694$$

$e \approx 2.718281828$

111. $f(x)=a^x$

$$\frac{f(x+h)-f(x)}{h}=\frac{a^{x+h}-a^x}{h}=\frac{a^x a^h-a^x}{h}=\frac{a^x\left(a^h-1\right)}{h}=a^x\left(\frac{a^h-1}{h}\right)$$

112. $f(x)=a^x$

$f(A+B)=a^{A+B}=a^A\cdot a^B=f(A)\cdot f(B)$

113. $f(x)=a^x$

$f(-x)=a^{-x}=\dfrac{1}{a^x}=\dfrac{1}{f(x)}$

114. $f(x)=a^x$

$f(\alpha x)=a^{\alpha x}=\left(a^x\right)^{\alpha}=[f(x)]^{\alpha}$

115. $\sinh x=\frac{1}{2}\left(e^x-e^{-x}\right)$

a. $f(-x)=\sinh(-x)$

$=\frac{1}{2}\left(e^{-x}-e^{x}\right)$

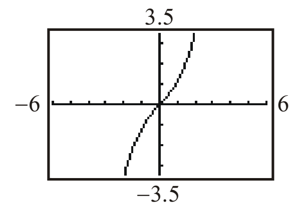
$=-\frac{1}{2}\left(e^{x}-e^{-x}\right)$

$=-\sinh x$

$=-f(x)$

Therefore, $f(x)=\sinh x$ is an odd function.

b. Let $Y_1=\frac{1}{2}\left(e^x-e^{-x}\right)$.



116. $\cosh x=\frac{1}{2}\left(e^x+e^{-x}\right)$

a. $f(-x)=\cosh(-x)$

$=\frac{1}{2}\left(e^{-x}+e^{x}\right)$

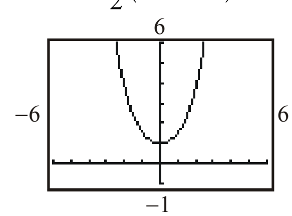
$=\frac{1}{2}\left(e^{x}+e^{-x}\right)$

$=\cosh x$

$=f(x)$

Thus, $f(x)=\cosh x$ is an even function.

b. Let $Y_1=\frac{1}{2}\left(e^x+e^{-x}\right)$.



c. $(\cosh x)^2 - (\sinh x)^2$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$
$$= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4}$$
$$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4}$$
$$= \frac{4}{4}$$
$$= 1$$

117. $f(x) = 2^{\left(2^x\right)} + 1$

$f(1) = 2^{\left(2^1\right)} + 1 = 2^2 + 1 = 4 + 1 = 5$

$f(2) = 2^{\left(2^2\right)} + 1 = 2^4 + 1 = 16 + 1 = 17$

$f(3) = 2^{\left(2^3\right)} + 1 = 2^8 + 1 = 256 + 1 = 257$

$f(4) = 2^{\left(2^4\right)} + 1 = 2^{16} + 1 = 65,536 + 1 = 65,537$

$f(5) = 2^{\left(2^5\right)} + 1 = 2^{32} + 1 = 4,294,967,296 + 1$
$= 4,294,967,297$
$= 641 \times 6,700,417$

118. Since the number of bacteria doubles every minute, half of the container is full one minute before it is full. Thus, it takes 59 minutes to fill the container.

119. Answers will vary.

120. Answers will vary.

121. Given the function $f(x) = a^x$, with $a > 1$,
If $x > 0$, the graph becomes steeper as a increases.
If $x < 0$, the graph becomes less steep as a increases.

122. Using the laws of exponents, we have:
$a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x$. So $y = a^{-x}$ and $y = \left(\frac{1}{a}\right)^x$ will have the same graph.

Section 4.4

1. $3x - 7 \le 8 - 2x$
$5x \le 15$
$x \le 3$
The solution set is $\{x \mid x \le 3\}$.

2. $x^2 - x - 6 > 0$
$(x-3)(x+2) > 0$
$f(x) = x^2 - x - 6$
$x = -2$ and $x = 3$ are the zeros of *f*.

Interval	$(-\infty, 2)$	$(-2, 3)$	$(3, \infty)$
Test Value	–3	0	4
Value of *f*	6	–6	6
Conclusion	positive	negative	positive

The solution set is $\{x \mid x < -2 \text{ or } x > 3\}$.

3. $\frac{x-1}{x+4} > 0$
$f(x) = \frac{x-1}{x+4}$
f is zero or undefined when $x = 1$ or $x = -4$.

Interval	$(-\infty, -4)$	$(-4, 1)$	$(1, \infty)$
Test Value	–5	0	2
Value of *f*	6	$-\frac{1}{4}$	$\frac{1}{6}$
Conclusion	positive	negative	positive

The solution set is $\{x \mid x < -4 \text{ or } x > 1\}$.

4. $\{x \mid x > 0\}$ or $(0, \infty)$

5. $(1, 0)$, $(a, 1)$, $\left(\frac{1}{a}, -1\right)$

6. 1

7. False. If $y = \log_a x$, then $x = a^y$.

8. True

9. $9 = 3^2$ is equivalent to $2 = \log_3 9$.

10. $16 = 4^2$ is equivalent to $2 = \log_4 16$.

11. $a^2 = 1.6$ is equivalent to $2 = \log_a 1.6$.

12. $a^3 = 2.1$ is equivalent to $3 = \log_a 2.1$.

13. $2^x = 7.2$ is equivalent to $x = \log_2 7.2$.

14. $3^x = 4.6$ is equivalent to $x = \log_3 4.6$.

15. $e^x = 8$ is equivalent to $x = \ln 8$.

16. $e^{2.2} = M$ is equivalent to $2.2 = \ln M$.

17. $\log_2 8 = 3$ is equivalent to $2^3 = 8$.

18. $\log_3\left(\frac{1}{9}\right) = -2$ is equivalent to $3^{-2} = \frac{1}{9}$.

19. $\log_a 3 = 6$ is equivalent to $a^6 = 3$.

20. $\log_b 4 = 2$ is equivalent to $b^2 = 4$.

21. $\log_3 2 = x$ is equivalent to $3^x = 2$.

22. $\log_2 6 = x$ is equivalent to $2^x = 6$.

23. $\ln 4 = x$ is equivalent to $e^x = 4$.

24. $\ln x = 4$ is equivalent to $e^4 = x$.

25. $\log_2 1 = 0$ since $2^0 = 1$.

26. $\log_8 8 = 1$ since $8^1 = 8$.

27. $\log_5 25 = 2$ since $5^2 = 25$.

28. $\log_3\left(\frac{1}{9}\right) = -2$ since $3^{-2} = \frac{1}{9}$.

29. $\log_{1/2} 16 = -4$ since $\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$.

30. $\log_{1/3} 9 = -2$ since $\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$.

31. $\log_{10}\sqrt{10} = \frac{1}{2}$ since $10^{1/2} = \sqrt{10}$.

32. $\log_5 \sqrt[3]{25} = \frac{2}{3}$ since $5^{2/3} = 25^{1/3} = \sqrt[3]{25}$.

33. $\log_{\sqrt{2}} 4 = 4$ since $\left(\sqrt{2}\right)^4 = 4$.

34. $\log_{\sqrt{3}} 9 = 4$ since $\left(\sqrt{3}\right)^4 = 9$.

35. $\ln\sqrt{e} = \frac{1}{2}$ since $e^{1/2} = \sqrt{e}$.

36. $\ln e^3 = 3$ since $e^3 = e^3$.

37. $f(x) = \ln(x-3)$ requires $x - 3 > 0$.

$$x - 3 > 0$$
$$x > 3$$

The domain of f is $\{x \mid x > 3\}$ or $(3, \infty)$.

38. $g(x) = \ln(x-1)$ requires $x - 1 > 0$.

$$x - 1 > 0$$
$$x > 1$$

The domain of g is $\{x \mid x > 1\}$ or $(1, \infty)$.

39. $F(x) = \log_2 x^2$ requires $x^2 > 0$.

$x^2 > 0$ for all $x \neq 0$.

The domain of F is $\{x \mid x \neq 0\}$.

40. $H(x) = \log_5 x^3$ requires $x^3 > 0$.

$x^3 > 0$ for all $x > 0$.

The domain of H is $\{x \mid x > 0\}$ or $(0, \infty)$.

41. $f(x) = 3 - 2\log_4\left[\frac{x}{2} - 5\right]$ requires $\frac{x}{2} - 5 > 0$.

$$\frac{x}{2} - 5 > 0$$
$$\frac{x}{2} > 5$$
$$x > 10$$

The domain of *f* is $\{x \mid x > 10\}$ or $(10, \infty)$.

42. $g(x)=8+5\ln(2x+3)$ requires $2x+3>0$.

$$2x+3>0$$
$$2x>-3$$
$$x>-\frac{3}{2}$$

The domain of g is $\left\{x \middle| x>-\frac{3}{2}\right\}$ or $\left(-\frac{3}{2},\infty\right)$.

43. $f(x)=\ln\left(\frac{1}{x+1}\right)$ requires $\frac{1}{x+1}>0$.

$p(x)=\frac{1}{x+1}$ is undefined when $x=-1$.

Interval	$(-\infty,-1)$	$(-1,\infty)$
Test Value	-2	0
Value of p	-1	1
Conclusion	negative	positive

The domain of f is $\{x \mid x>-1\}$ or $(-1,\infty)$.

44. $g(x)=\ln\left(\frac{1}{x-5}\right)$ requires $\frac{1}{x-5}>0$.

$p(x)=\frac{1}{x-5}$ is undefined when $x=5$.

Interval	$(-\infty,5)$	$(5,\infty)$
Test Value	4	6
Value of p	-1	1
Conclusion	negative	positive

The domain of g is $\{x \mid x>5\}$ or $(5,\infty)$.

45. $g(x)=\log_5\left(\frac{x+1}{x}\right)$ requires $\frac{x+1}{x}>0$.

$p(x)=\frac{x+1}{x}$ is zero or undefined when $x=-1$ or $x=0$.

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
Test Value	-2	$-\frac{1}{2}$	1
Value of p	$\frac{1}{2}$	-1	2
Conclusion	positive	negative	positive

The domain of g is $\{x \mid x<-1 \text{ or } x>0\}$; $(-\infty,-1)$ or $(0,\infty)$.

46. $h(x)=\log_3\left(\frac{x}{x-1}\right)$ requires $\frac{x}{x-1}>0$.

$p(x)=\frac{x}{x-1}$ is zero or undefined when $x=0$ or $x=1$.

Interval	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
Test Value	-1	$-\frac{1}{2}$	2
Value of p	$\frac{1}{2}$	-1	2
Conclusion	positive	negative	positive

The domain of h is $\{x \mid x<0 \text{ or } x>1\}$; $(-\infty,0)$ or $(1,\infty)$.

47. $f(x)=\sqrt{\ln x}$ requires $\ln x\ge 0$ and $x>0$

$$\ln x\ge 0$$
$$x\ge e^0$$
$$x\ge 1$$

The domain of h is $\{x \mid x\ge 1\}$ or $[1,\infty)$.

48. $g(x)=\frac{1}{\ln x}$ requires $\ln x\ne 0$ and $x>0$

$$\ln x\ne 0$$
$$x\ne e^0$$
$$x\ne 1$$

The domain of h is $\{x \mid x>0 \text{ and } x\ne 1\}$; $(0,1)$ or $(1,\infty)$.

49. $\ln\left(\frac{5}{3}\right)\approx 0.511$

50. $\frac{\ln(5)}{3}\approx 0.536$

51. $\frac{\ln\frac{10}{3}}{0.04}\approx 30.099$

52. $\frac{\ln\frac{2}{3}}{-0.1}\approx 4.055$

53. $\frac{\ln 4+\ln 2}{\log 4+\log 2}\approx 2.303$

54. $\dfrac{\log 15+\log 20}{\ln 15+\ln 20}\approx 0.434$

55. $\dfrac{2\ln 5+\log 50}{\log 4-\ln 2}\approx -53.991$

56. $\dfrac{3\log 80-\ln 5}{\log 5+\ln 20}\approx 1.110$

57. If the graph of $f(x)=\log_a x$ contains the point $(2,2)$, then $f(2)=\log_a 2=2$. Thus,

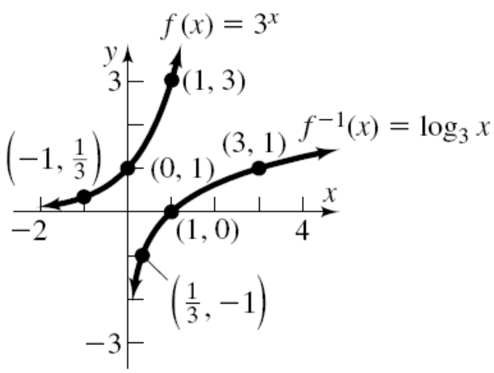
$$\log_a 2 = 2$$
$$a^2 = 2$$
$$a = \pm\sqrt{2}$$

Since the base a must be positive by definition, we have that $a=\sqrt{2}$.

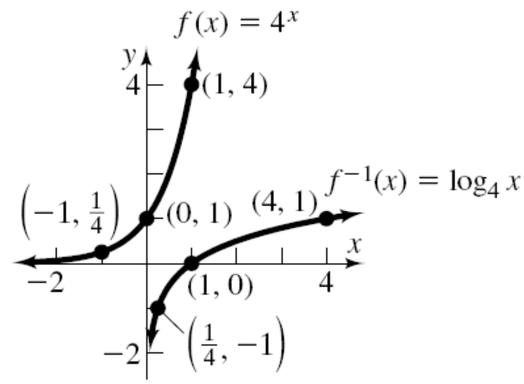
58. If the graph of $f(x)=\log_a x$ contains the point $\left(\dfrac{1}{2},-4\right)$, then $f\left(\dfrac{1}{2}\right)=\log_a\left(\dfrac{1}{2}\right)=-4$. Thus,

$$\log_a\left(\frac{1}{2}\right)=-4$$
$$a^{-4}=\frac{1}{2}$$
$$\frac{1}{a^4}=\frac{1}{2}$$
$$a^4=2$$
$$a=2^{1/4}\approx 1.189$$

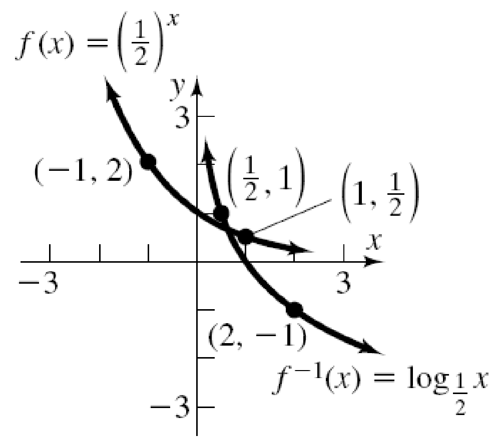
59.



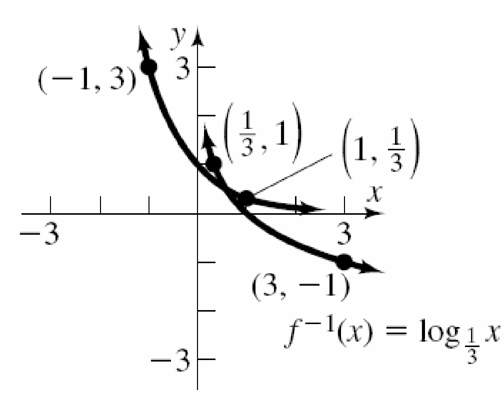
60.



61.



62.



63. B

64. F

65. D

66. H

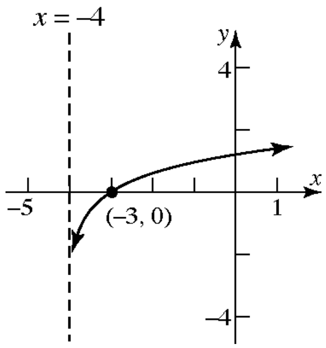
67. A

68. C

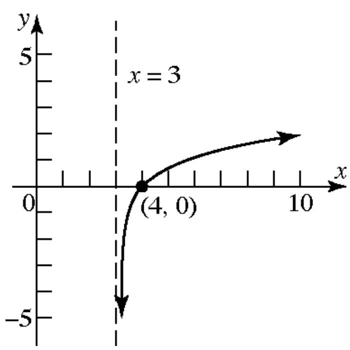
69. E

70. G

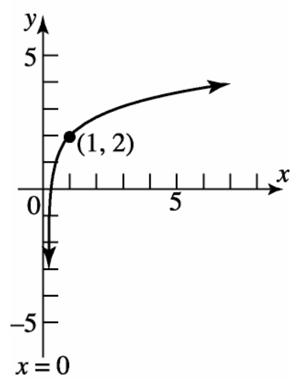
71. $f(x) = \ln(x+4)$
Using the graph of $y = \ln x$, shift the graph 4 units to the left.
Domain: $(-4, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -4$



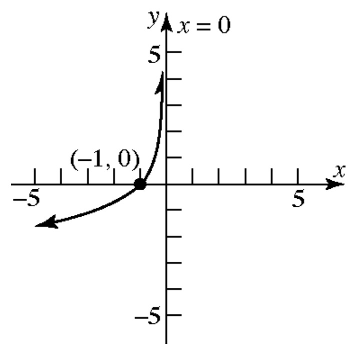
72. $f(x) = \ln(x-3)$
Using the graph of $y = \ln x$, shift the graph 3 units to the right.
Domain: $(3, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 3$



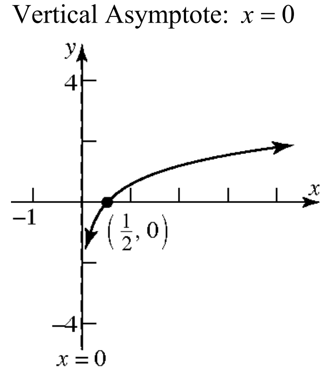
73. $f(x) = 2 + \ln x = \ln x + 2$
Using the graph of $y = \ln x$, shift up 2 units.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



74. $f(x) = -\ln(-x)$
Using the graph of $y = \ln x$, reflect the graph about the y-axis, and reflect about the x-axis.
Domain: $(-\infty, 0)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



75. $g(x) = \ln(2x)$
Using the graph of $y = \ln x$, compress the graph horizontally by a factor of $\frac{1}{2}$.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



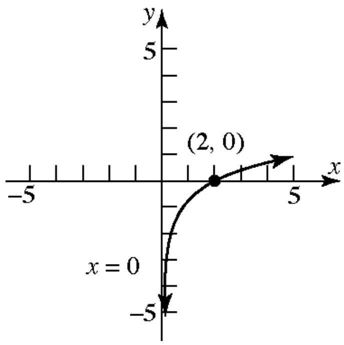
76. $h(x) = \ln\left(\frac{1}{2}x\right)$

Using the graph of $y = \ln x$, stretch the graph horizontally by a factor of 2.

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$



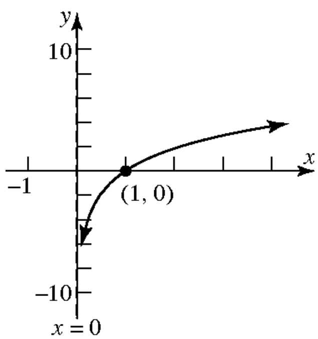
77. $f(x) = 3\ln x$

Using the graph of $y = \ln x$, stretch the graph vertically by a factor of 3.

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$



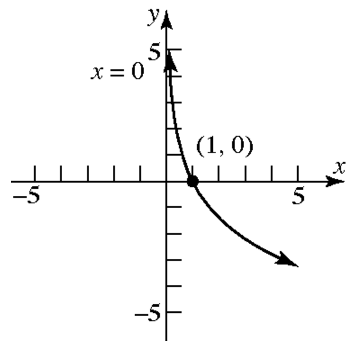
78. $f(x) = -2\ln x$

Using the graph of $y = \ln x$, stretch the graph vertically by a factor of 2, and reflect about the x-axis.

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$



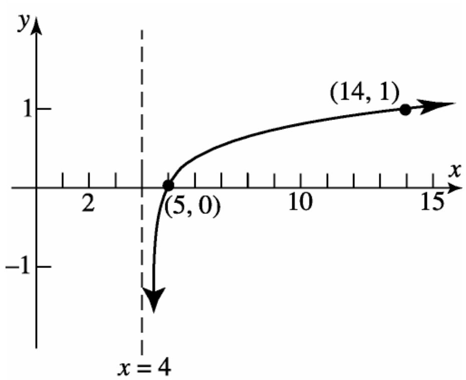
79. $f(x) = \log(x-4)$

Using the graph of $y = \log x$, shift 4 units to the right.

Domain: $(4, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 4$



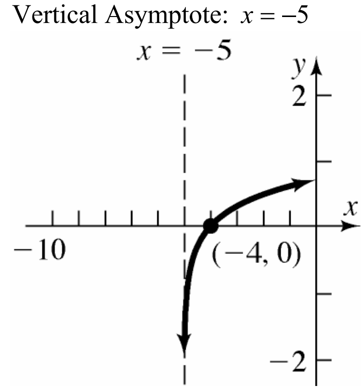
80. $f(x) = \log(x+5)$

Using the graph of $y = \log x$, shift 5 units to the left.

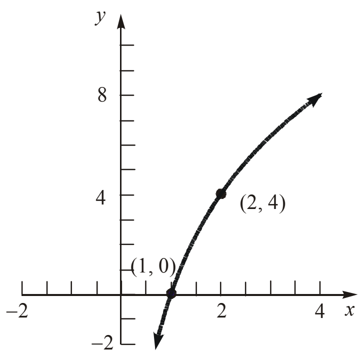
Domain: $(-5, \infty)$

Range: $(-\infty, \infty)$

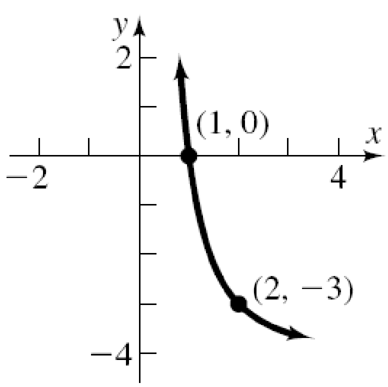
Vertical Asymptote: $x = -5$



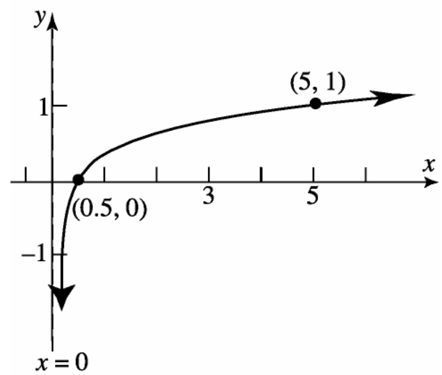
81. $h(x) = 4\log_2 x$
Using the graph of $y = \log_2 x$, stretch the graph vertically by a factor of 4.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



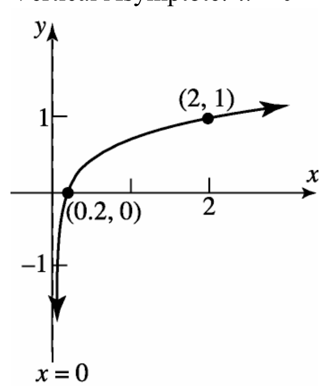
82. $g(x) = -3\log_2 x$
Using the graph of $y = \log_2 x$, reflect the graph about the x-axis, and stretch the graph vertically by a factor of 3.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



83. $F(x) = \log(2x)$
Using the graph of $y = \log x$, compress the graph horizontally by a factor of $\frac{1}{2}$.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



84. $g(x) = \log(5x)$
Using the graph of $y = \log x$, compress the graph horizontally by a factor of $\frac{1}{5}$.
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$



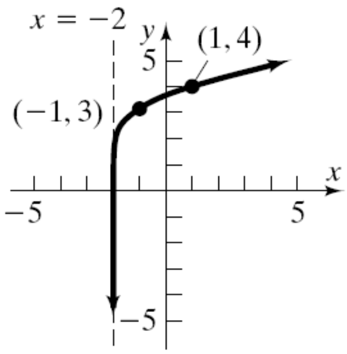
85. $f(x) = 3 + \log_3(x+2) = \log_3(x+2) + 3$

Using the graph of $y = \log_3 x$, shift 2 units to the left, and shift up 3 units.

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = -2$



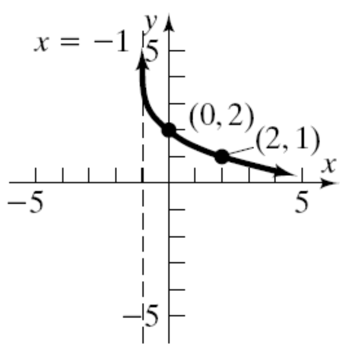
86. $f(x) = 2 - \log_3(x+1) = -\log_3(x+1) + 2$

Using the graph of $y = \log_3 x$, shift 1 unit to the left, reflect the graph about the x-axis, and shift 2 units up.

Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = -1$



87. $\log_3 x = 2$

$$x = 3^2$$
$$x = 9$$

The solution set is $\{9\}$.

88. $\log_5 x = 3$

$$x = 5^3$$
$$x = 125$$

The solution set is $\{125\}$.

89. $\log_2(2x+1) = 3$

$$2x + 1 = 2^3$$
$$2x + 1 = 8$$
$$2x = 7$$
$$x = \frac{7}{2}$$

The solution set is $\left\{\frac{7}{2}\right\}$.

90. $\log_3(3x-2) = 2$

$$3x - 2 = 3^2$$
$$3x - 2 = 9$$
$$3x = 11$$
$$x = \frac{11}{3}$$

The solution set is $\left\{\frac{11}{3}\right\}$.

91. $\log_x 4 = 2$

$$x^2 = 4$$
$$x = 2 \quad (x \neq -2, \text{ base is positive})$$

The solution set is $\{2\}$.

92. $\log_x\left(\frac{1}{8}\right) = 3$

$$x^3 = \frac{1}{8}$$
$$x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}\right\}$.

93. $\ln e^x = 5$

$$e^x = e^5$$
$$x = 5$$

The solution set is $\{5\}$.

94. $\ln e^{-2x} = 8$

$$e^{-2x} = e^8$$
$$-2x = 8$$
$$x = -4$$

The solution set is $\{-4\}$.

95. $\log_4 64 = x$

$$4^x = 64$$
$$4^x = 4^3$$
$$x = 3$$

The solution set is $\{3\}$.

96. $\log_5 625 = x$

$$5^x = 625$$
$$5^x = 5^4$$
$$x = 4$$

The solution set is $\{4\}$.

97. $\log_3 243 = 2x+1$

$$3^{2x+1} = 243$$
$$3^{2x+1} = 3^5$$
$$2x+1 = 5$$
$$2x = 4$$
$$x = 2$$

The solution set is $\{2\}$.

98. $\log_6 36 = 5x+3$

$$6^{5x+3} = 36$$
$$6^{5x+3} = 6^2$$
$$5x+3 = 2$$
$$5x = -1$$
$$x = -\frac{1}{5}$$

The solution set is $\left\{-\frac{1}{5}\right\}$.

99. $e^{3x} = 10$

$$3x = \ln 10$$
$$x = \frac{\ln 10}{3}$$

The solution set is $\left\{\frac{\ln 10}{3}\right\}$.

100. $e^{-2x} = \frac{1}{3}$

$$-2x = \ln\left(\frac{1}{3}\right)$$
$$-2x = \ln\left(3^{-1}\right)$$
$$-2x = -\ln 3$$
$$2x = \ln 3$$
$$x = \frac{\ln 3}{2}$$

The solution set is $\left\{\frac{\ln 3}{2}\right\}$.

101. $e^{2x+5} = 8$

$$2x+5 = \ln 8$$
$$2x = -5+\ln 8$$
$$x = \frac{-5+\ln 8}{2}$$

The solution set is $\left\{\frac{-5+\ln 8}{2}\right\}$.

102. $e^{-2x+1} = 13$

$$-2x+1 = \ln 13$$
$$-2x = -1+\ln 13$$
$$x = \frac{-1+\ln 13}{-2} = \frac{1-\ln(13)}{2}$$

The solution set is $\left\{\frac{1-\ln(13)}{2}\right\}$.

103. $\log_3\left(x^2+1\right) = 2$

$$x^2+1 = 3^2$$
$$x^2+1 = 9$$
$$x^2 = 8$$
$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

The solution set is $\left\{-2\sqrt{2},\ 2\sqrt{2}\right\}$.

104. $\log_5\left(x^2+x+4\right)=2$

$$x^2+x+4=5^2$$
$$x^2+x+4=25$$
$$x^2+x-21=0$$
$$x=\frac{-1\pm\sqrt{1^2-4(1)(-21)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{85}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{85}}{2},\ \frac{-1+\sqrt{85}}{2}\right\}$.

105. $\log_2 8^x=-3$

$$8^x=2^{-3}$$
$$\left(2^3\right)^x=2^{-3}$$
$$2^{3x}=2^{-3}$$
$$3x=-3$$
$$x=-1$$

The solution set is $\{-1\}$.

106. $\log_3 3^x=-1$

$$3^x=3^{-1}$$
$$x=-1$$

The solution set is $\{-1\}$.

107. a. $G(x)=\log_3(2x+1)$

We require that $2x+1$ be positive.

$$2x+1>0$$
$$2x>-1$$
$$x>-\frac{1}{2}$$

Domain: $\left\{x \mid x>-\frac{1}{2}\right\}$ or $\left(-\frac{1}{2},\infty\right)$

b. $G(40)=\log_3(2\cdot 40+1)$

$$=\log_3 81$$
$$=4$$

The point $(40,4)$ is on the graph of G.

c. $G(x)=2$

$$\log_3(2x+1)=2$$
$$2x+1=3^2$$
$$2x+1=9$$
$$2x=8$$
$$x=4$$

The point $(4,2)$ is on the graph of G.

108. a. $F(x)=\log_2(x+1)-3$

We require that $x+1$ be positive.

$$x+1>0$$
$$x>-1$$

Domain: $\{x \mid x>-1\}$ or $(-1,\infty)$

b. $F(7)=\log_2(7+1)-3$

$$=\log_2(8)-3$$
$$=3-3$$
$$=0$$

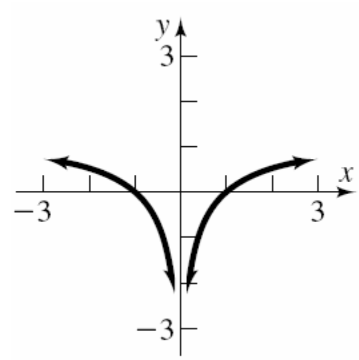
The point $(7,0)$ is on the graph of F.

c. $F(x)=-1$

$$\log_2(x+1)-3=-1$$
$$\log_2(x+1)=2$$
$$x+1=2^2$$
$$x+1=4$$
$$x=3$$

The point $(3,-1)$ is on the graph of F.

109. $f(x)=\begin{cases}\ln(-x) & \text{if } x<0\\ \ln x & \text{if } x>0\end{cases}$

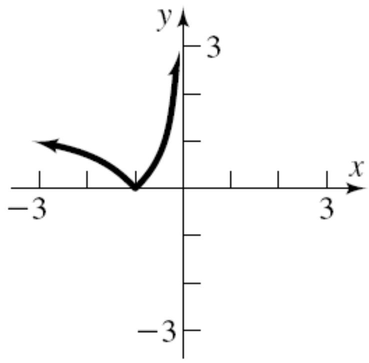


Domain: $\{x \mid x\neq 0\}$

Range: all real numbers; $(-\infty,\infty)$

Intercepts: $(-1,0)$, $(1,0)$

110. $f(x)=\begin{cases}\ln(-x) & \text{if } x \le -1\\ -\ln(-x) & \text{if } -1 < x < 0\end{cases}$

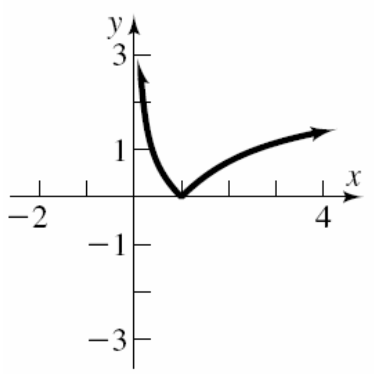


Domain: $\{x \mid x < 0\}$; $(-\infty, 0)$

Range: $\{y \mid y \ge 0\}$; $[-,\infty)$

Intercept: $(-1, 0)$

111. $f(x)=\begin{cases}-\ln x & \text{if } 0 < x < 1\\ \ln x & \text{if } x \ge 1\end{cases}$

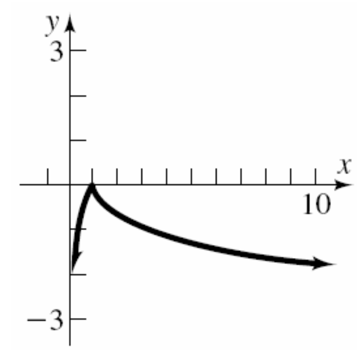


Domain: $\{x \mid x > 0\}$; $(0, \infty)$

Range: $\{y \mid y \ge 0\}$; $[0, \infty)$

Intercept: $(1, 0)$

112. $f(x)=\begin{cases}\ln x & \text{if } 0 < x < 1\\ -\ln x & \text{if } x \ge 1\end{cases}$



Domain: $\{x \mid x > 0\}$; $(0, \infty)$

Range: $\{y \mid y \le 0\}$; $(-\infty, 0]$

Intercept: $(1, 0)$

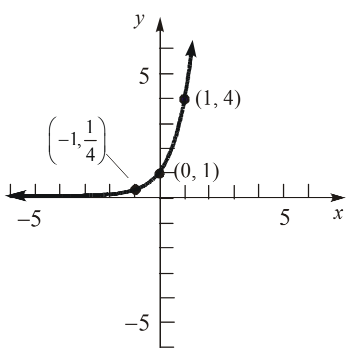
113.
$$\begin{aligned} f(x) &= e^{2x+3}\\ y &= e^{2x+3}\\ x &= e^{2y+3}\\ \ln x &= 2y+3\\ \ln x - 3 &= 2y\\ \frac{\ln x - 3}{2} &= y \quad \text{(inverse)}\\ f^{-1}(x) &= \frac{\ln x - 3}{2} \end{aligned}$$

114.
$$\begin{aligned} f(x) &= 3^{x-5}+4\\ y &= 3^{x-5}+4\\ x &= 3^{y-5}+4\\ x-4 &= 3^{y-5}\\ \log_3(x-4) &= y-5\\ \log_3(x-4)+5 &= y \quad \text{(inverse)}\\ f^{-1}(x) &= \log_3(x-4)+5 \end{aligned}$$

115.
$$\begin{aligned} f(x) &= \log_2(5x)-3\\ y &= \log_2(5x)-3\\ x &= \log_2(5y)-3\\ x+3 &= \log_2(5y)\\ 2^{x+3} &= 5y\\ \frac{1}{5}\cdot 2^{x+3} &= y \quad \text{(inverse)}\\ f^{-1}(x) &= \frac{1}{5}\cdot 2^{x+3} \end{aligned}$$

116.
$$f(x)=\log_5(x+1)+4$$
$$y=\log_5(x+1)+4$$
$$x=\log_5(y+1)+4$$
$$x-4=\log_5(y+1)$$
$$5^{x-4}=y+1$$
$$5^{x-4}-1=y \quad \text{(inverse)}$$
$$f^{-1}(x)=5^{x-4}-1$$

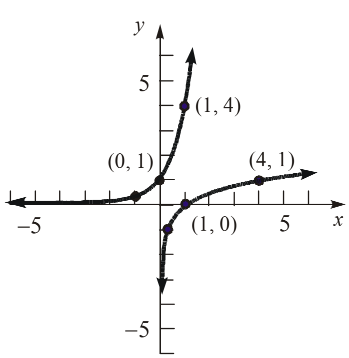
117. a. Graphing $f(x)=4^x$:



Domain: $(-\infty,\ \infty)$
Range: $(0,\ \infty)$
Horizontal asymptote: $y=0$

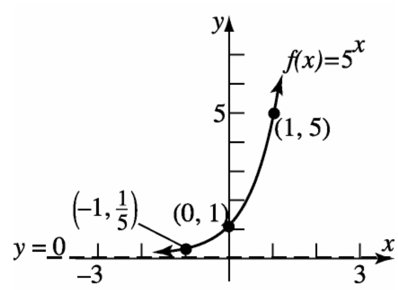
b. Finding the inverse:
$$f(x)=4^x$$
$$y=4^x$$
$$x=4^y \quad \text{Inverse}$$
$$y=\log_4 x$$
$$f^{-1}(x)=\log_4 x$$

c. Graphing the inverse:



Domain: $(0,\ \infty)$
Range: $(-\infty,\ \infty)$
Vertical asymptote: $x=0$

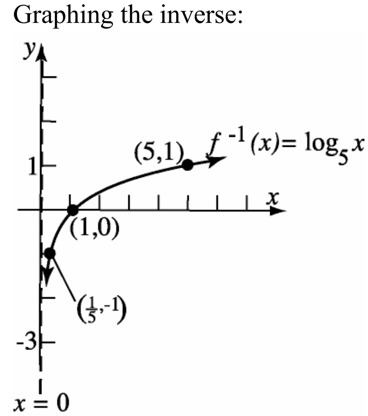
118. a. Graphing $f(x)=5^x$:



Domain: $(-\infty,\ \infty)$
Range: $(0,\ \infty)$
Horizontal asymptote: $y=0$

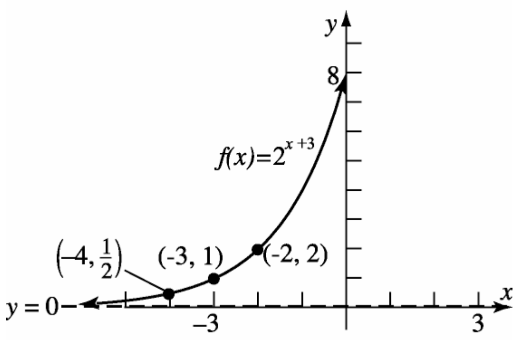
b. Finding the inverse:
$$f(x)=5^x$$
$$y=5^x$$
$$x=5^y \quad \text{Inverse}$$
$$y=\log_5 x$$
$$f^{-1}(x)=\log_5 x$$

c. Graphing the inverse:



Domain: $(0,\ \infty)$
Range: $(-\infty,\ \infty)$
Vertical asymptote: $x=0$

119. a. Graphing $f(x) = 2^{x+3}$:

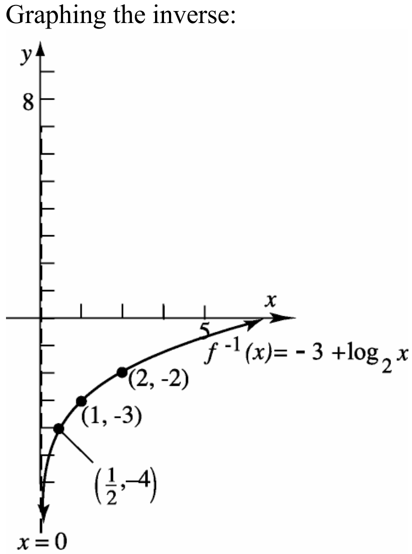


Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Horizontal asymptote: $y = 0$

b. Finding the inverse:

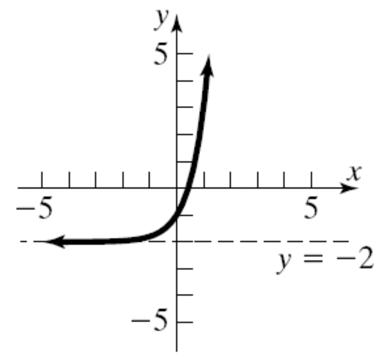
$$f(x) = 2^{x+3}$$
$$y = 2^{x+3}$$
$$x = 2^{y+3} \quad \text{Inverse}$$
$$y + 3 = \log_2 x$$
$$y = -3 + \log_2 x$$
$$f^{-1}(x) = -3 + \log_2 x$$

c. Graphing the inverse:



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$

120. a. Graphing $f(x) = 3^x - 2$:

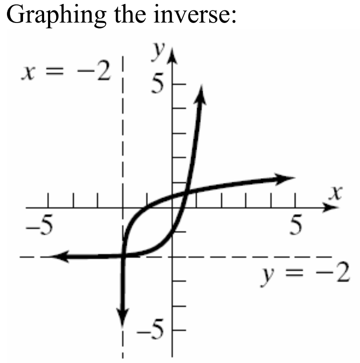


Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
Horizontal asymptote: $y = -2$

b. Finding the inverse:

$$f(x) = 3^x - 2$$
$$y = 3^x - 2$$
$$x = 3^y - 2 \quad \text{Inverse}$$
$$x + 2 = 3^y$$
$$y = \log_3(x+2)$$
$$f^{-1}(x) = \log_3(x+2)$$

c. Graphing the inverse:



Domain: $(-2, \infty)$
Range: $(-\infty, \infty)$
Vertical asymptote: $x = -2$

121. $\text{pH} = -\log_{10}\left[\text{H}^+\right]$

a. $\text{pH} = -\log_{10}\left[0.1\right] = -(-1) = 1$

b. $\text{pH} = -\log_{10}\left[0.01\right] = -(-2) = 2$

c. $\text{pH} = -\log_{10}\left[0.001\right] = -(-3) = 3$

d. As the H^+ decreases, the pH increases.

e.
$$\begin{aligned} 3.5 &= -\log_{10}\left[H^+\right] \\ -3.5 &= \log_{10}\left[H^+\right] \\ \left[H^+\right] &= 10^{-3.5} \\ &\approx 3.16\times 10^{-4} \\ &= 0.000316 \end{aligned}$$

f.
$$\begin{aligned} 7.4 &= -\log_{10}\left[H^+\right] \\ -7.4 &= \log_{10}\left[H^+\right] \\ \left[H^+\right] &= 10^{-7.4} \\ &\approx 3.981\times 10^{-8} \\ &= 0.00000003981 \end{aligned}$$

122.
$$\begin{aligned} H &= -\left(p_1 \log p_1 + p_2 \log p_2 + \cdots + p_n \log p_n\right) \\ &= -p_1 \log p_1 - p_2 \log p_2 - \cdots - p_n \log p_n \end{aligned}$$

a.
$$\begin{aligned} H &= -0.014\log(0.014) - 0.041\log(0.041) \\ &\quad - 0.124\log(0.124) - 0.128\log(0.128) \\ &\quad - 0.003\log(0.003) - 0.690\log(0.690) \\ &\approx 0.4283 \end{aligned}$$

b. $H_{\max} = \log(6) \approx 0.7782$

c. $E = \dfrac{H}{H_{\max}} \approx 0.5504$

123. $p = 760e^{-0.145h}$

a.
$$\begin{aligned} 320 &= 760e^{-0.145h} \\ \frac{320}{760} &= e^{-0.145h} \\ \ln\left(\frac{320}{760}\right) &= -0.145h \\ h &= \frac{\ln\left(\frac{320}{760}\right)}{-0.145} \approx 5.97 \end{aligned}$$

Approximately 5.97 kilometers.

b.
$$\begin{aligned} 667 &= 760e^{-0.145h} \\ \frac{667}{760} &= e^{-0.145h} \\ \ln\left(\frac{667}{760}\right) &= -0.145h \\ h &= \frac{\ln\left(\frac{667}{760}\right)}{-0.145} \approx 0.90 \end{aligned}$$

Approximately 0.90 kilometers.

124. $A = A_0 e^{-0.35n}$

a.
$$\begin{aligned} 50 &= 100e^{-0.35n} \\ 0.5 &= e^{-0.35n} \\ \ln(0.5) &= -0.35n \\ t &= \frac{\ln(0.5)}{-0.35} \approx 1.98 \end{aligned}$$

Approximately 2 days.

b.
$$\begin{aligned} 10 &= 100e^{-0.35n} \\ 0.1 &= e^{-0.35n} \\ \ln(0.1) &= -0.35n \\ t &= \frac{\ln(0.1)}{-0.35} \approx 6.58 \end{aligned}$$

About 6.58 days, or 6 days and 14 hours.

125. $F(t) = 1 - e^{-0.1t}$

a.
$$\begin{aligned} 0.5 &= 1 - e^{-0.1t} \\ -0.5 &= -e^{-0.1t} \\ 0.5 &= e^{-0.1t} \\ \ln(0.5) &= -0.1t \\ t &= \frac{\ln(0.5)}{-0.1} \approx 6.93 \end{aligned}$$

Approximately 6.93 minutes.

b.
$$\begin{aligned} 0.8 &= 1 - e^{-0.1t} \\ -0.2 &= -e^{-0.1t} \\ 0.2 &= e^{-0.1t} \\ \ln(0.2) &= -0.1t \\ t &= \frac{\ln(0.2)}{-0.1} \approx 16.09 \end{aligned}$$

Approximately 16.09 minutes.

c. It is impossible for the probability to reach 100% because $e^{-0.1t}$ will never equal zero; thus, $F(t) = 1 - e^{-0.1t}$ will never equal 1.

126. $F(t)=1-e^{-0.15t}$

a.
$$0.50=1-e^{-0.15t}$$
$$-0.5=e^{-(R/L)t}$$
$$0.5=e^{-0.15t}$$
$$\ln(0.5)=-0.15t$$
$$t=\frac{\ln(0.5)}{-0.15}\approx 4.62$$

Approximately 4.62 minutes, or 4 minutes and 37 seconds.

b.
$$0.80=1-e^{-0.15t}$$
$$-0.2=-e^{-0.15t}$$
$$0.2=e^{-0.15t}$$
$$\ln(0.2)=-0.15t$$
$$t=\frac{\ln(0.2)}{-0.15}\approx 10.73$$

Approximately 10.73 minutes, or 10 minutes and 44 seconds.

127.
$$D=5e^{-0.4h}$$
$$2=5e^{-0.4h}$$
$$0.4=e^{-0.4h}$$
$$\ln(0.4)=-0.4h$$
$$h=\frac{\ln(0.4)}{-0.4}\approx 2.29$$

Approximately 2.29 hours, or 2 hours and 17 minutes.

128.
$$N=P\left(1-e^{-0.15d}\right)$$
$$450=1000\left(1-e^{-0.15d}\right)$$
$$0.45=1-e^{-0.15d}$$
$$-0.55=-e^{-0.15d}$$
$$0.55=e^{-0.15d}$$
$$\ln(0.55)=-0.15d$$
$$d=\frac{\ln(0.55)}{-0.15}\approx 3.99$$

Approximately 4 days.

129. $I=\frac{E}{R}\left[1-e^{-(R/L)t}\right]$

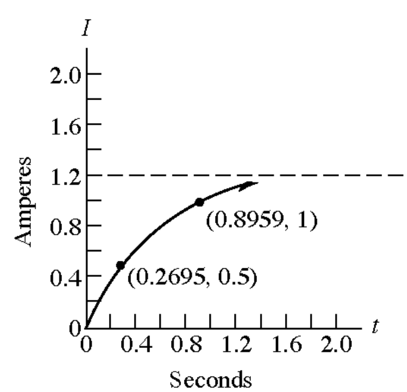
Substituting $E=12$, $R=10$, $L=5$, and $I=0.5$, we obtain:
$$0.5=\frac{12}{10}\left[1-e^{-(10/5)t}\right]$$
$$\frac{5}{12}=1-e^{-2t}$$
$$e^{-2t}=\frac{7}{12}$$
$$-2t=\ln(7/12)$$
$$t=\frac{\ln(7/12)}{-2}\approx 0.2695$$

It takes approximately 0.2695 second to obtain a current of 0.5 ampere.

Substituting $E=12$, $R=10$, $L=5$, and $I=1.0$, we obtain:
$$1.0=\frac{12}{10}\left[1-e^{-(10/5)t}\right]$$
$$\frac{10}{12}=1-e^{-2t}$$
$$e^{-2t}=\frac{1}{6}$$
$$-2t=\ln(1/6)$$
$$t=\frac{\ln(1/6)}{-2}\approx 0.8959$$

It takes approximately 0.8959 second to obtain a current of 0.5 ampere.

Graphing:



130. $L(t) = A\left(1-e^{-kt}\right)$

a. $20 = 200\left(1-e^{-k(5)}\right)$

$0.1 = 1-e^{-5k}$

$e^{-5k} = 0.9$

$-5k = \ln(0.9)$

$k = \dfrac{\ln(0.9)}{-5} \approx 0.0211$

b. $L(10) = 200\left(1-e^{-0.021(10)}\right)$

$= 200\left(1-e^{-0.21}\right)$

≈ 38 words

c. $L(15) = 200\left(1-e^{-0.021(15)}\right)$

$= 200\left(1-e^{-0.315}\right)$

≈ 54 words

d. $180 = 200\left(1-e^{-0.021t}\right)$

$0.9 = 1-e^{-0.021t}$

$e^{-0.021t} = 0.1$

$-0.021t = \ln(0.1)$

$t = \dfrac{\ln(0.1)}{-0.021} \approx 109$ minutes

131. $L\left(10^{-7}\right) = 10\log\left(\dfrac{10^{-7}}{10^{-12}}\right)$

$= 10\log\left(10^{5}\right)$

$= 10\cdot 5$

$= 50$ decibels

132. $L\left(10^{-3}\right) = 10\log\left(\dfrac{10^{-3}}{10^{-12}}\right)$

$= 10\log\left(10^{9}\right)$

$= 10\cdot 9$

$= 90$ decibels

133. $L\left(10^{-1}\right) = 10\log\left(\dfrac{10^{-1}}{10^{-12}}\right)$

$= 10\log\left(10^{11}\right)$

$= 10\cdot 11$

$= 110$ decibels

134. Intensity of car:

$70 = 10\log\left(\dfrac{x}{10^{-12}}\right)$

$7 = \log\left(\dfrac{x}{10^{-12}}\right)$

$10^{7} = \dfrac{x}{10^{-12}}$

$x = 10^{-5}$

Intensity of truck is $10\cdot 10^{-5} = 10^{-4}$.

$L\left(10^{-4}\right) = 10\log\left(\dfrac{10^{-4}}{10^{-12}}\right)$

$= 10\log\left(10^{8}\right)$

$= 10\cdot 8$

$= 80$ decibels

135. $M(125{,}892) = \log\left(\dfrac{125{,}892}{10^{-3}}\right) \approx 8.1$

136. $M(7943) = \log\left(\dfrac{7943}{10^{-3}}\right) \approx 6.9$

137. $R = e^{kx}$

a. $1.4 = e^{k(0.03)}$

$1.4 = e^{0.03k}$

$\ln(1.4) = 0.03k$

$k = \dfrac{\ln(1.4)}{0.03} \approx 11.216$

b. $R = e^{11.216(0.17)} = e^{1.90672} \approx 6.73$

c. $100 = e^{11.216x}$

$100 = e^{11.216x}$

$\ln(100) = 11.216x$

$x = \dfrac{\ln(100)}{11.216} \approx 0.41$

d. $5 = e^{11.216x}$

$\ln 5 = 11.216x$

$x = \dfrac{\ln 5}{11.216} \approx 0.143$

At a percent concentration of 0.143 or higher, the driver should be charged with a DUI.

e. Answers will vary.

138. No. Explanations will vary.

139. If the base of a logarithmic function equals 1, we would have the following:

$f(x) = \log_1(x)$

$f^{-1}(x) = 1^x = 1$ for every real number x.

In other words, f^{-1} would be a constant function and, therefore, f^{-1} would not be one-to-one.

140. $\text{New} = \text{Old}\left(e^{Rt}\right)$

Age	Depriciation rate
1	$38{,}000 = 36{,}600e^{R\,(1)}$ $\frac{38{,}000}{36{,}600} = e^{R}$ $R = \ln\left(\frac{38{,}000}{36{,}600}\right) \approx 0.03754 \approx 3.8\%$
2	$38{,}000 = 32{,}400e^{R\,(2)}$ $\frac{38{,}000}{32{,}400} = e^{2R}$ $\ln\left(\frac{38{,}000}{32{,}400}\right) = 2R$ $R = \frac{\ln\left(\frac{38{,}000}{32{,}400}\right)}{2} \approx 0.07971 \approx 8\%$
3	$38{,}000 = 28{,}750e^{R\,(3)}$ $\frac{38{,}000}{28{,}750} = e^{3R}$ $\ln\left(\frac{38{,}000}{28{,}750}\right) = 3R$ $R = \frac{\ln\left(\frac{38{,}000}{28{,}750}\right)}{3} \approx 0.0930 \approx 9.3\%$
4	$38{,}000 = 25{,}400e^{R\,(4)}$ $\frac{38{,}000}{25{,}400} = e^{4R}$ $\ln\left(\frac{38{,}000}{25{,}400}\right) = 4R$ $R = \frac{\ln\left(\frac{38{,}000}{24{,}500}\right)}{4} \approx 0.1007 \approx 10.1\%$

Age	Depriciation rate
5	$38{,}000 = 21{,}200e^{R\,(5)}$ $\frac{38{,}000}{21{,}200} = e^{5R}$ $\ln\left(\frac{38{,}000}{21{,}200}\right) = 5R$ $R = \frac{\ln\left(\frac{38{,}000}{21{,}200}\right)}{5} \approx 0.1167 \approx 11.7\%$

Answers will vary.

Section 4.5

1. sum

2. 7

3. $r\log_a M$

4. False

5. False

6. True

7. $\log_3 3^{71} = 71$

8. $\log_2 2^{-13} = -13$

9. $\ln e^{-4} = -4$

10. $\ln e^{\sqrt{2}} = \sqrt{2}$

11. $2^{\log_2 7} = 7$

12. $e^{\ln 8} = 8$

13. $\log_8 2 + \log_8 4 = \log_8(4\cdot 2) = \log_8 8 = 1$

14.
$$\begin{aligned}\log_6 9 + \log_6 4 &= \log_6(9\cdot 4)\\ &= \log_6 36\\ &= \log_6 6^2\\ &= 2\end{aligned}$$

15. $\log_6 18 - \log_6 3 = \log_6 \frac{18}{3} = \log_6 6 = 1$

16. $\log_8 16 - \log_8 2 = \log_8 \frac{16}{2} = \log_8 8 = 1$

17.
$$\begin{aligned}\log_2 6 \cdot \log_6 4 &= \log_6 4^{\log_2 6}\\ &= \log_6 \left(2^2\right)^{\log_2 6}\\ &= \log_6 2^{2\log_2 6}\\ &= \log_6 2^{\log_2 6^2}\\ &= \log_6 6^2\\ &= 2\end{aligned}$$

18.
$$\begin{aligned}\log_3 8 \cdot \log_8 9 &= \log_8 9^{\log_3 8}\\ &= \log_8 \left(3^2\right)^{\log_3 8}\\ &= \log_8 3^{2\log_3 8}\\ &= \log_8 3^{\log_3 8^2}\\ &= \log_8 8^2\\ &= 2\end{aligned}$$

19. $3^{\log_3 5 - \log_3 4} = 3^{\log_3 \frac{5}{4}} = \frac{5}{4}$

20. $5^{\log_5 6 + \log_5 7} = 5^{\log_5 (6\cdot 7)} = 5^{\log_5 42} = 42$

21. $e^{\log_{e^2} 16}$

Let $a = \log_{e^2} 16$, then $\left(e^2\right)^a = 16$.

$$\begin{aligned}e^{2a} &= 16\\ e^{2a} &= 4^2\\ \left(e^{2a}\right)^{1/2} &= \left(4^2\right)^{1/2}\\ e^a &= 4\\ a &= \ln 4\end{aligned}$$

Thus, $e^{\log_{e^2} 16} = e^{\ln 4} = 4$.

22. $e^{\log_{e^2} 9}$

Let $a = \log_{e^2} 9$, then $\left(e^2\right)^a = 9$.

$$\begin{aligned}e^{2a} &= 9\\ e^{2a} &= 3^2\\ \left(e^{2a}\right)^{1/2} &= \left(3^2\right)^{1/2}\\ e^a &= 3\\ a &= \ln 3\end{aligned}$$

Thus, $e^{\log_{e^2} 9} = e^{\ln 3} = 3$.

23. $\ln 6 = \ln(2\cdot 3) = \ln 2 + \ln 3 = a + b$

24. $\ln \frac{2}{3} = \ln 2 - \ln 3 = a - b$

25. $\ln 1.5 = \ln \frac{3}{2} = \ln 3 - \ln 2 = b - a$

26. $\ln 0.5 = \ln \frac{1}{2} = \ln 1 - \ln 2 = 0 - a = -a$

27. $\ln 8 = \ln 2^3 = 3\cdot \ln 2 = 3a$

28. $\ln 27 = \ln 3^3 = 3 \cdot \ln 3 = 3b$

29.
$$\begin{aligned}\ln \sqrt[5]{6} &= \ln 6^{1/5}\\ &= \frac{1}{5}\ln 6\\ &= \frac{1}{5}\ln(2\cdot 3)\\ &= \frac{1}{5}(\ln 2 + \ln 3)\\ &= \frac{1}{5}(a+b)\end{aligned}$$

30.
$$\begin{aligned}\ln \sqrt[4]{\frac{2}{3}} &= \ln\left(\frac{2}{3}\right)^{1/4}\\ &= \frac{1}{4}\ln\frac{2}{3}\\ &= \frac{1}{4}(\ln 2 - \ln 3)\\ &= \frac{1}{4}(a-b)\end{aligned}$$

31. $\log_5(25x) = \log_5 25 + \log_5 x = 2 + \log_5 x$

32. $\log_3 \frac{x}{9} = \log_3 \frac{x}{3^2} = \log_3 x - \log_3 3^2 = \log_3 x - 2$

33. $\log_2 z^3 = 3\log_2 z$

34. $\log_7 x^5 = 5\log_7 x$

35. $\ln(ex) = \ln e + \ln x = 1 + \ln x$

36. $\ln \frac{e}{x} = \ln e - \ln x = 1 - \ln x$

37. $\ln\left(xe^x\right) = \ln x + \ln e^x = \ln x + x$

38. $\ln\left(\frac{x}{e^x}\right) = \ln x - \ln e^x = \ln x - x$

39. $\log_a\left(u^2v^3\right) = \log_a u^2 + \log_a v^3$
$= 2\log_a u + 3\log_a v$

40. $\log_2\left(\frac{a}{b^2}\right) = \log_2 a - \log_2 b^2 = \log_2 a - 2\log_2 b$

41. $\ln\left(x^2\sqrt{1-x}\right) = \ln x^2 + \ln\sqrt{1-x}$
$= \ln x^2 + \ln(1-x)^{1/2}$
$= 2\ln x + \frac{1}{2}\ln(1-x)$

42. $\ln\left(x\sqrt{1+x^2}\right) = \ln x + \ln\sqrt{1+x^2}$
$= \ln x + \ln\left(1+x^2\right)^{1/2}$
$= \ln x + \frac{1}{2}\ln\left(1+x^2\right)$

43. $\log_2\left(\frac{x^3}{x-3}\right) = \log_2 x^3 - \log_2(x-3)$
$= 3\log_2 x - \log_2(x-3)$

44. $\log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right)$
$= \log_5\left(x^2+1\right)^{1/3} - \log_5(x^2-1)$
$= \frac{1}{3}\log_5\left(x^2+1\right) - \log_5\left(x^2-1\right)$
$= \frac{1}{3}\log_5\left(x^2+1\right) - \log_5\left((x-1)(x+1)\right)$
$= \frac{1}{3}\log_5\left(x^2+1\right) - \log_5(x-1) - \log_5(x+1)$

45. $\log\left[\frac{x(x+2)}{(x+3)^2}\right] = \log\left[x(x+2)\right] - \log(x+3)^2$
$= \log x + \log(x+2) - 2\log(x+3)$

46. $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right] = \log\left(x^3\sqrt{x+1}\right) - \log(x-2)^2$
$= \log x^3 + \log(x+1)^{1/2} - 2\log(x-2)$
$= 3\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)$

47. $\ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{1/3}$
$= \frac{1}{3}\ln\left[\frac{(x-2)(x+1)}{(x+4)^2}\right]$
$= \frac{1}{3}\left[\ln(x-2)(x+1) - \ln(x+4)^2\right]$
$= \frac{1}{3}\left[\ln(x-2) + \ln(x+1) - 2\ln(x+4)\right]$
$= \frac{1}{3}\ln(x-2) + \frac{1}{3}\ln(x+1) - \frac{2}{3}\ln(x+4)$

48. $\ln\left[\frac{(x-4)^2}{x^2-1}\right]^{2/3}$
$= \frac{2}{3}\ln\left[\frac{(x-4)^2}{x^2-1}\right]$
$= \frac{2}{3}\left[\ln(x-4)^2 - \ln\left(x^2-1\right)\right]$
$= \frac{2}{3}\left[2\ln(x-4) - \ln\left((x-1)(x+1)\right)\right]$
$= \frac{2}{3}\left[2\ln(x-4) - \ln(x-1) - \ln(x+1)\right]$
$= \frac{4}{3}\ln(x-4) - \frac{2}{3}\ln(x-1) - \frac{2}{3}\ln(x+1)$

49. $\ln\frac{5x\sqrt{1+3x}}{(x-4)^3}$
$= \ln\left(5x\sqrt{1+3x}\right) - \ln(x-4)^3$
$= \ln 5 + \ln x + \ln\sqrt{1+3x} - 3\ln(x-4)$
$= \ln 5 + \ln x + \ln(1+3x)^{1/2} - 3\ln(x-4)$
$= \ln 5 + \ln x + \frac{1}{2}\ln(1+3x) - 3\ln(x-4)$

50. $\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right]$
$= \ln\left(5x^2\sqrt[3]{1-x}\right) - \ln\left(4(x+1)^2\right)$
$= \ln 5 + \ln x^2 + \ln(1-x)^{1/3} - \left[\ln 4 + \ln(x+1)^2\right]$
$= \ln 5 + 2\ln x + \frac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)$

51. $3\log_5 u + 4\log_5 v = \log_5 u^3 + \log_5 v^4$
$= \log_5\left(u^3v^4\right)$

52. $2\log_3 u - \log_3 v = \log_3 u^2 - \log_3 v = \log_3\left(\dfrac{u^2}{v}\right)$

53. $\log_3 \sqrt{x} - \log_3 x^3 = \log_3\left(\dfrac{\sqrt{x}}{x^3}\right)$
$= \log_3\left(\dfrac{x^{1/2}}{x^3}\right)$
$= \log_3 x^{-5/2}$
$= -\dfrac{5}{2}\log_3 x$

54. $\log_2\left(\dfrac{1}{x}\right) + \log_2\left(\dfrac{1}{x^2}\right) = \log_2\left(\dfrac{1}{x}\cdot\dfrac{1}{x^2}\right)$
$= \log_2\left(\dfrac{1}{x^3}\right)$
$= \log_2 x^{-3}$
$= -3\log_2 x$

55. $\log_4\left(x^2-1\right) - 5\log_4\left(x+1\right)$
$= \log_4\left(x^2-1\right) - \log_4\left(x+1\right)^5$
$= \log_4\left(\dfrac{x^2-1}{\left(x+1\right)^5}\right)$
$= \log_4\left(\dfrac{\left(x+1\right)\left(x-1\right)}{\left(x+1\right)^5}\right)$
$= \log_4\left(\dfrac{x-1}{\left(x+1\right)^4}\right)$

56. $\log\left(x^2+3x+2\right) - 2\log_2\left(x+1\right)$
$= \log\left(x^2+3x+2\right) - \log_2\left(x+1\right)^2$
$= \log\left(\dfrac{x^2+3x+2}{\left(x+1\right)^2}\right)$
$= \log\left(\dfrac{\left(x+2\right)\left(x+1\right)}{\left(x+1\right)^2}\right)$
$= \log\left(\dfrac{x+2}{x+1}\right)$

57. $\ln\left(\dfrac{x}{x-1}\right) + \ln\left(\dfrac{x+1}{x}\right) - \ln\left(x^2-1\right)$
$= \ln\left[\dfrac{x}{x-1}\cdot\dfrac{x+1}{x}\right] - \ln\left(x^2-1\right)$
$= \ln\left[\dfrac{x+1}{x-1} \div \left(x^2-1\right)\right]$
$= \ln\left[\dfrac{x+1}{\left(x-1\right)\left(x^2-1\right)}\right]$
$= \ln\left[\dfrac{x+1}{(x-1)(x-1)(x+1)}\right]$
$= \ln\left(\dfrac{1}{(x-1)^2}\right)$
$= \ln(x-1)^{-2}$
$= -2\ln(x-1)$

58. $\log\left(\dfrac{x^2+2x-3}{x^2-4}\right) - \log\left(\dfrac{x^2+7x+6}{x+2}\right)$
$= \log\left[\dfrac{\left(\dfrac{x^2+2x-3}{x^2-4}\right)}{\left(\dfrac{x^2+7x+6}{x+2}\right)}\right]$
$= \log\left[\dfrac{(x+3)(x-1)}{(x-2)(x+2)}\cdot\dfrac{x+2}{(x+6)(x+1)}\right]$
$= \log\left[\dfrac{(x+3)(x-1)}{(x-2)(x+6)(x+1)}\right]$

59. $8\log_2\sqrt{3x-2} - \log_2\left(\dfrac{4}{x}\right) + \log_2 4$
$= \log_2\left(\sqrt{3x-2}\right)^8 - \left(\log_2 4 - \log_2 x\right) + \log_2 4$
$= \log_2(3x-2)^4 - \log_2 4 + \log_2 x + \log_2 4$
$= \log_2(3x-2)^4 + \log_2 x$
$= \log_2\left[x(3x-2)^4\right]$

60. $21\log_3\sqrt[3]{x} + \log_3\left(9x^2\right) - \log_3 9$
$= \log_3\left(x^{1/3}\right)^{21} + \log_3(9) + \log_3\left(x^2\right) - \log_3 9$
$= \log_3 x^7 + \log_3 x^2$
$= \log_3\left(x^7\cdot x^2\right)$
$= \log_3\left(x^9\right)$

61. $2\log_a\left(5x^3\right)-\frac{1}{2}\log_a(2x+3)$
$=\log_a\left(5x^3\right)^2-\log_a(2x-3)^{1/2}$
$=\log_a\left(25x^6\right)-\log_a\sqrt{2x-3}$
$=\log_a\left[\frac{25x^6}{\sqrt{2x-3}}\right]$

62. $\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{2}\log\left(x^2+1\right)$
$=\log\left(x^3+1\right)^{1/3}+\log\left(x^2+1\right)^{1/2}$
$=\log\left[\sqrt[3]{x^3+1}\cdot\sqrt{x^2+1}\right]$

63. $2\log_2(x+1)-\log_2(x+3)-\log_2(x-1)$
$=\log_2(x+1)^2-\log_2(x+3)-\log_2(x-1)$
$=\log_2\frac{(x+1)^2}{(x+3)}-\log_2(x-1)$
$=\log_2\left(\frac{(x+1)^2}{(x+3)(x-1)}\right)$

64. $3\log_5(3x+1)-2\log_5(2x-1)-\log_5 x$
$=\log_5(3x+1)^3-\log_5(2x-1)^2-\log_5 x$
$=\log_5\frac{(3x+1)^3}{(2x-1)^2}-\log_5 x$
$=\log_5\frac{(3x+1)^3}{x(2x-1)^2}$

65. $\log_3 21=\frac{\log 21}{\log 3}\approx 2.771$

66. $\log_5 18=\frac{\log 18}{\log 5}\approx 1.796$

67. $\log_{1/3} 71=\frac{\log 71}{\log(1/3)}=\frac{\log 71}{-\log 3}\approx -3.880$

68. $\log_{1/2} 15=\frac{\log 15}{\log(1/2)}=\frac{\log 15}{-\log 2}\approx -3.907$

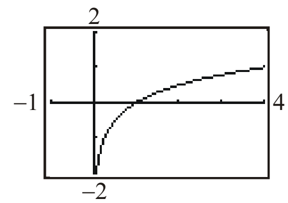
69. $\log_{\sqrt{2}} 7=\frac{\log 7}{\log\sqrt{2}}\approx 5.615$

70. $\log_{\sqrt{5}} 8=\frac{\log 8}{\log\sqrt{5}}\approx 2.584$

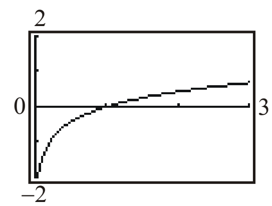
71. $\log_{\pi} e=\frac{\ln e}{\ln\pi}\approx 0.874$

72. $\log_{\pi}\sqrt{2}=\frac{\ln\sqrt{2}}{\ln\pi}\approx 0.303$

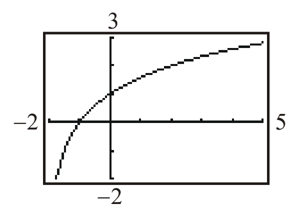
73. $y=\log_4 x=\frac{\ln x}{\ln 4}$ or $y=\frac{\log x}{\log 4}$



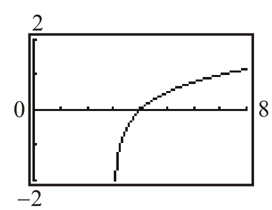
74. $y=\log_5 x=\frac{\ln x}{\ln 5}$ or $y=\frac{\log x}{\log 5}$



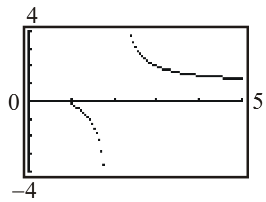
75. $y=\log_2(x+2)=\frac{\ln(x+2)}{\ln 2}$ or $y=\frac{\log(x+2)}{\log 2}$



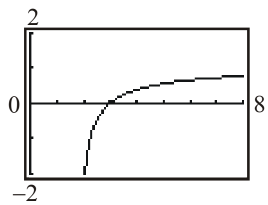
76. $y=\log_4(x-3)=\frac{\ln(x-3)}{\ln 4}$ or $y=\frac{\log(x-3)}{\log 4}$



77. $y = \log_{x-1}(x+1) = \dfrac{\ln(x+1)}{\ln(x-1)}$ or $y = \dfrac{\log(x+1)}{\log(x-1)}$



78. $y = \log_{x+2}(x-2) = \dfrac{\ln(x-2)}{\ln(x+2)}$ or $y = \dfrac{\log(x-2)}{\log(x+2)}$



79. $f(x) = \ln x$; $g(x) = e^x$; $h(x) = x^2$

a. $(f \circ g)(x) = f(g(x))$
$= \ln(e^x) = x$

Domain: $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

b. $(g \circ f)(x) = g(f(x))$
$= e^{\ln x} = x$
Domain: $\{x \mid x > 0\}$ or $(0, \infty)$
(note: the restriction on the domain is due to the domain of $\ln x$)

c. $(f \circ g)(5) = 5$ [from part (a)]

d. $(f \circ h)(x) = f(h(x)) = \ln(x^2)$

e. $(f \circ h)(e) = \ln(e^2) = 2\ln e = 2 \cdot 1 = 2$

80. $f(x) = \log_2 x$; $g(x) = 2^x$; $h(x) = 4x$

a. $(f \circ g)(x) = f(g(x))$
$= \log_2(2^x) = x$

Domain: $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

b. $(g \circ f)(x) = g(f(x))$
$= 2^{\log_2 x} = x$
Domain: $\{x \mid x > 0\}$ or $(0, \infty)$
(note: the restriction on the domain is due to the domain of $\log_2 x$)

c. $(f \circ g)(3) = 3$ [from part (a)]

d. $(f \circ h)(x) = f(h(x))$
$= \log_2(4x)$
or
$= \log_2 4 + \log_2 x$
$= 2 + \log_2 x$

e. $(f \circ h)(8) = \log_2(4 \cdot 8) = \log_2 32 = 5$
or
$= 2 + \log_2 8 = 2 + 3 = 5$

81. $\ln y = \ln x + \ln C$
$\ln y = \ln(xC)$
$y = Cx$

82. $\ln y = \ln(x + C)$
$y = x + C$

83. $\ln y = \ln x + \ln(x+1) + \ln C$
$\ln y = \ln(x(x+1)C)$
$y = Cx(x+1)$

84. $\ln y = 2\ln x - \ln(x+1) + \ln C$
$\ln y = \ln\left(\dfrac{x^2 C}{x+1}\right)$
$y = \dfrac{Cx^2}{x+1}$

85. $\ln y = 3x + \ln C$
$\ln y = \ln e^{3x} + \ln C$
$\ln y = \ln(Ce^{3x})$
$y = Ce^{3x}$

86. $\ln y = -2x + \ln C$

$\ln y = \ln e^{-2x} + \ln C$

$\ln y = \ln\left(Ce^{-2x}\right)$

$y = Ce^{-2x}$

87. $\ln(y-3) = -4x + \ln C$

$\ln(y-3) = \ln e^{-4x} + \ln C$

$\ln(y-3) = \ln\left(Ce^{-4x}\right)$

$y - 3 = Ce^{-4x}$

$y = Ce^{-4x} + 3$

88. $\ln(y+4) = 5x + \ln C$

$\ln(y+4) = \ln e^{5x} + \ln C$

$\ln(y+4) = \ln\left(Ce^{5x}\right)$

$y + 4 = Ce^{5x}$

$y = Ce^{5x} - 4$

89. $3\ln y = \frac{1}{2}\ln(2x+1) - \frac{1}{3}\ln(x+4) + \ln C$

$\ln y^3 = \ln(2x+1)^{1/2} - \ln(x+4)^{1/3} + \ln C$

$\ln y^3 = \ln\left[\frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}\right]$

$y^3 = \frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}$

$y = \left[\frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}\right]^{1/3}$

$y = \frac{\sqrt[3]{C}\,(2x+1)^{1/6}}{(x+4)^{1/9}}$

90. $2\ln y = -\frac{1}{2}\ln x + \frac{1}{3}\ln\left(x^2+1\right) + \ln C$

$\ln y^2 = -\ln x^{1/2} + \ln\left(x^2+1\right)^{1/3} + \ln C$

$\ln y^2 = \ln\left[\frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}\right]$

$y^2 = \frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}$

$y = \left[\frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}\right]^{1/2}$

$y = \frac{\sqrt{C}\left(x^2+1\right)^{1/6}}{x^{1/4}}$

91. $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$

$= \frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} \cdot \frac{\log 5}{\log 4} \cdot \frac{\log 6}{\log 5} \cdot \frac{\log 7}{\log 6} \cdot \frac{\log 8}{\log 7}$

$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$

$= \frac{3\log 2}{\log 2}$

$= 3$

92. $\log_2 4 \cdot \log_4 6 \cdot \log_6 8 = \frac{\log 4}{\log 2} \cdot \frac{\log 6}{\log 4} \cdot \frac{\log 8}{\log 6}$

$= \frac{\log 8}{\log 2}$

$= \frac{\log 2^3}{\log 2}$

$= \frac{3\log 2}{\log 2}$

$= 3$

93. $\log_2 3 \cdot \log_3 4 \cdots \log_n (n+1) \cdot \log_{n+1} 2$

$= \frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} \cdots \frac{\log(n+1)}{\log n} \cdot \frac{\log 2}{\log(n+1)}$

$= \frac{\log 2}{\log 2}$

$= 1$

94. $\log_2 2 \cdot \log_2 4 \cdot \ldots \cdot \log_2 2^n$
$= \log_2 2 \cdot \log_2 2^2 \cdots \log_2 2^n$
$= 1 \cdot 2 \cdot 3 \cdots n$
$= n!$

95. $\log_a\left(x+\sqrt{x^2-1}\right)+\log_a\left(x-\sqrt{x^2-1}\right)$:
$= \log_a\left[\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)\right]$
$= \log_a\left[x^2-\left(x^2-1\right)\right]$
$= \log_a\left[x^2-x^2+1\right]$
$= \log_a 1$
$= 0$

96. $\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)$
$= \log_a\left[\left(\sqrt{x}+\sqrt{x-1}\right)\left(\sqrt{x}-\sqrt{x-1}\right)\right]$
$= \log_a\left[x-\left(x-1\right)\right]$
$= \log_a\left[x-x+1\right]$
$= \log_a 1$
$= 0$

97. $2x+\ln\left(1+e^{-2x}\right) = \ln e^{2x}+\ln\left(1+e^{-2x}\right)$
$= \ln\left(e^{2x}\left(1+e^{-2x}\right)\right)$
$= \ln\left(e^{2x}+e^{0}\right)$
$= \ln\left(e^{2x}+1\right)$

98. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\log_a(x+h)-\log_a x}{h}$
$= \dfrac{\log_a\left(\dfrac{x+h}{x}\right)}{h}$
$= \dfrac{1}{h}\cdot\log_a\left(1+\dfrac{h}{x}\right)$
$= \log_a\left(1+\dfrac{h}{x}\right)^{\frac{1}{h}}, \quad h \neq 0$

99. $f(x) = \log_a x$ means that $x = a^{f(x)}$.
Now, raising both sides to the -1 power, we obtain $x^{-1} = \left(a^{f(x)}\right)^{-1} = \left(a^{-1}\right)^{f(x)} = \left(\dfrac{1}{a}\right)^{f(x)}$.
$x^{-1} = \left(\dfrac{1}{a}\right)^{f(x)}$ means that $\log_{1/a} x^{-1} = f(x)$.
Thus, $\log_{1/a} x^{-1} = f(x)$
$-\log_{1/a} x = f(x)$
$-f(x) = \log_{1/a} x$

100. $f(AB) = \log_a(AB)$
$= \log_a A + \log_a B$
$= f(A)+f(B)$

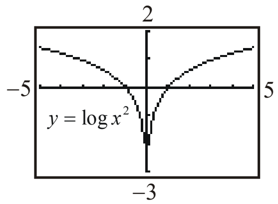
101. $f(x) = \log_a x$
$f\left(\dfrac{1}{x}\right) = \log_a\left(\dfrac{1}{x}\right)$
$= \log_a 1 - \log_a x$
$= -\log_a x$
$= -f(x)$

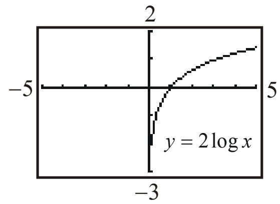
102. $f\left(x\right) = \log_a x$
$f\left(x^{\alpha}\right) = \log_a x^{\alpha} = \alpha \log_a x = \alpha f(x)$

103. If $A = \log_a M$ and $B = \log_a N$, then $a^A = M$ and $a^B = N$.
$\log_a\left(\dfrac{M}{N}\right) = \log_a\left(\dfrac{a^A}{a^B}\right)$
$= \log_a a^{A-B}$
$= A-B$
$= \log_a M - \log_a N$

104. $\log_a\left(\dfrac{1}{N}\right) = \log_a N^{-1}$
$= -1\cdot\log_a N$
$= -\log_a N, \quad a \neq 1$

105. $Y_1 = \log x^2$ $Y_2 = 2\log x$





The domain of $Y_1 = \log_a x^2$ is $\{x \mid x \neq 0\}$. The domain of $Y_2 = 2\log_a x$ is $\{x \mid x > 0\}$. These two domains are different because the logarithm property $\log_a x^n = n \cdot \log_a x$ holds only when $\log_a x$ exists.

106. Answers may vary. One possibility follows: Let $x = 4$ and $y = 4$. Then $\log_2(x+y) = \log_2(4+4) = \log_2 8 = 3$. But $\log_2 x + \log_2 y = \log_2 4 + \log_2 4 = 2 + 2 = 4$. Thus, $\log_2(4+4) \neq \log_2 4 + \log_2 4$ and, in general, $\log_2(x+y) \neq \log_2 x + \log_2 y$.

107. Answers may vary. One possibility follows: Let $a = 2$, $x = 8$, and $r = 3$. Then $(\log_a x)^r = (\log_2 8)^3 = 3^3 = 27$. But $r\log_a x = 3\log_2 8 = 3 \cdot 3 = 9$. Thus, $(\log_2 8)^3 \neq 3\log_2 8$ and, in general, $(\log_a x)^r \neq r\log_a x$.

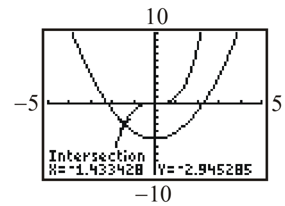
108. No. $\log_3(-5)$ does not exist.

Section 4.6

1.
$$x^2 - 7x - 30 = 0$$
$$(x+3)(x-10) = 0$$
$$x+3 = 0 \quad \text{or} \quad x-10 = 0$$
$$x = -3 \quad \text{or} \quad x = 10$$
The solution set is $\{-3, 10\}$.

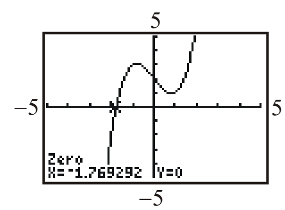
2. Let $u = x+3$. Then
$$(x+3)^2 - 4(x+3) + 3 = 0$$
$$u^2 - 4u + 3 = 0$$
$$(u-1)(u-3) = 0$$
$$u-1 = 0 \quad \text{or} \quad u-3 = 0$$
$$u = 1 \quad \text{or} \quad u = 3$$
Back substituting $u = x+3$, we obtain
$$x+3 = 1 \quad \text{or} \quad x+3 = 3$$
$$x = -2 \quad \text{or} \quad x = 0$$
The solution set is $\{-2, 0\}$.

3. $x^3 = x^2 - 5$
Using INTERSECT to solve:
$y_1 = x^3$; $y_2 = x^2 - 5$



Thus, $x \approx -1.43$, so the solution set is $\{-1.43\}$.

4. $x^3 - 2x + 2 = 0$
Using ZERO to solve: $y_1 = x^3 - 2x + 2$



Thus, $x \approx -1.77$, so the solution set is $\{-1.77\}$.

5.
$$\log_4(x+2) = \log_4 8$$
$$x+2 = 8$$
$$x = 6$$
The solution set is $\{6\}$.

6.
$$\log_5(2x+3) = \log_5 3$$
$$2x+3 = 3$$
$$2x = 0$$
$$x = 0$$
The solution set is $\{0\}$.

7.
$$\frac{1}{2}\log_3 x = 2\log_3 2$$
$$\log_3 x^{1/2} = \log_3 2^2$$
$$x^{1/2} = 4$$
$$x = 16$$
The solution set is $\{16\}$.

8.
$$\begin{aligned} -2\log_4 x &= \log_4 9 \\ \log_4 x^{-2} &= \log_4 9 \\ x^{-2} &= 9 \\ \frac{1}{x^2} &= 9 \\ x^2 &= \frac{1}{9} \\ x &= \pm\frac{1}{3} \end{aligned}$$

Since $\log_4\left(-\frac{1}{3}\right)$ is undefined, the solution set is $\left\{\frac{1}{3}\right\}$.

9.
$$\begin{aligned} 2\log_5 x &= 3\log_5 4 \\ \log_5 x^2 &= \log_5 4^3 \\ x^2 &= 64 \\ x &= \pm 8 \end{aligned}$$

Since $\log_5(-8)$ is undefined, the solution set is $\{8\}$.

10.
$$\begin{aligned} 3\log_2 x &= -\log_2 27 \\ \log_2 x^3 &= \log_2 27^{-1} \\ x^3 &= 27^{-1} \\ x^3 &= \frac{1}{27} \\ x &= \frac{1}{3} \end{aligned}$$

The solution set is $\left\{\frac{1}{3}\right\}$.

11.
$$\begin{aligned} 3\log_2(x-1) + \log_2 4 &= 5 \\ \log_2(x-1)^3 + \log_2 4 &= 5 \\ \log_2\left(4(x-1)^3\right) &= 5 \\ 4(x-1)^3 &= 2^5 \\ (x-1)^3 &= \frac{32}{4} \\ (x-1)^3 &= 8 \\ x-1 &= 2 \\ x &= 3 \end{aligned}$$

The solution set is $\{3\}$.

12.
$$\begin{aligned} 2\log_3(x+4) - \log_3 9 &= 2 \\ \log_3(x+4)^2 - \log_3 3^2 &= 2 \\ \log_3(x+4)^2 - 2 &= 2 \\ \log_3(x+4)^2 &= 4 \\ (x+4)^2 &= 3^4 \\ (x+4)^2 &= 81 \\ x+4 &= \pm 9 \\ x &= -4 \pm 9 \\ x &= 5 \text{ or } x = -13 \end{aligned}$$

Since $\log_3(-13+4) = \log_3(-9)$ is undefined, the solution set is $\{5\}$.

13.
$$\begin{aligned} \log x + \log(x+15) &= 2 \\ \log\left(x(x+15)\right) &= 2 \\ x(x+15) &= 10^2 \\ x^2 + 15x - 100 &= 0 \\ (x+20)(x-5) &= 0 \\ x &= -20 \text{ or } x = 5 \end{aligned}$$

Since $\log(-20)$ is undefined, the solution set is $\{5\}$.

14.
$$\begin{aligned} \log_4 x + \log_4(x-3) &= 1 \\ \log_4\left(x(x-3)\right) &= 1 \\ x(x-3) &= 4^1 \\ x^2 - 3x - 4 &= 0 \\ (x+1)(x-4) &= 0 \\ x &= -1 \text{ or } x = 4 \end{aligned}$$

Since $\log_4(-1)$ is undefined, the solution set is $\{4\}$.

15. $\ln x + \ln(x+2) = 4$

$$\begin{aligned}
\ln\left(x(x+2)\right) &= 4\\
x(x+2) &= e^4\\
x^2 + 2x - e^4 &= 0\\
x &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-e^4)}}{2(1)}\\
&= \frac{-2 \pm \sqrt{4+4e^4}}{2}\\
&= \frac{-2 \pm 2\sqrt{1+e^4}}{2}\\
&= -1 \pm \sqrt{1+e^4}
\end{aligned}$$

$x = -1 - \sqrt{1+e^4}$ or $x = -1 + \sqrt{1+e^4}$

$\approx -8.456 \qquad \approx 6.456$

Since $\ln(-8.456)$ is undefined, the solution set is $\left\{-1+\sqrt{1+e^4} \approx 6.456\right\}$.

16. $\ln(x+1) - \ln x = 2$

$$\begin{aligned}
\ln\left(\frac{x+1}{x}\right) &= 2\\
\frac{x+1}{x} &= e^2\\
x+1 &= e^2 x\\
e^2 x - x &= 1\\
x\left(e^2-1\right) &= 1\\
x &= \frac{1}{e^2-1} \approx 0.157
\end{aligned}$$

The solution set is $\left\{\frac{1}{e^2-1} \approx 0.157\right\}$.

17. $\log_3(x+12) + \log_3(x+4) = 2$

$$\begin{aligned}
\log_3\left[(x+12)(x+4)\right] &= 2\\
(x+12)(x+4) &= 3^2\\
x^2 + 16x + 48 &= 9\\
x^2 + 16x + 39 &= 0\\
(x+13)(x+3) &= 0
\end{aligned}$$

$x+13 = 0$ or $x+3 = 0$

$x = -13 \qquad x = -3$

The solution set is $\{-3\}$. Note: -13 does not satisfy the original equation, so it must be excluded from the solution set.

18. $\log_2(x+11) + \log_2(x+7) = 5$

$$\begin{aligned}
\log_2\left[(x+11)(x+7)\right] &= 5\\
(x+11)(x+7) &= 2^5\\
x^2 + 18x + 77 &= 32\\
x^2 + 18x + 45 &= 0\\
(x+15)(x+3) &= 0
\end{aligned}$$

$x+15 = 0$ or $x+3 = 0$

$x = -15 \qquad x = -3$

The solution set is $\{-3\}$. Note: -15 does not satisfy the original equation, so it must be excluded from the solution set.

19. $2^{2x} + 2^x - 12 = 0$

$$\begin{aligned}
\left(2^x\right)^2 + 2^x - 12 &= 0\\
\left(2^x-3\right)\left(2^x+4\right) &= 0
\end{aligned}$$

$2^x - 3 = 0$ or $2^x + 4 = 0$

$2^x = 3$ or $2^x = -4$

$\log\left(2^x\right) = \log 3$ No solution

$x \log 2 = \log 3$

$x = \frac{\log 3}{\log 2} \approx 1.585$

The solution set is $\left\{\frac{\log 3}{\log 2} \approx 1.585\right\}$.

20. $3^{2x} + 3^x - 2 = 0$

$$\begin{aligned}
\left(3^x\right)^2 + 3^x - 2 &= 0\\
\left(3^x-1\right)\left(3^x+2\right) &= 0
\end{aligned}$$

$3^x - 1 = 0$ or $3^x + 2 = 0$

$3^x = 1$ or $3^x = -2$

$x = 0$ No solution

The solution set is $\{0\}$.

21. $3^{2x} + 3^{x+1} - 4 = 0$

$$\begin{aligned}
\left(3^x\right)^2 + 3 \cdot 3^x - 4 &= 0\\
\left(3^x-1\right)\left(3^x+4\right) &= 0
\end{aligned}$$

$3^x - 1 = 0$ or $3^x + 4 = 0$

$3^x = 1$ or $3^x = -4$

$x = 0$ No solution

The solution set is $\{0\}$.

22.
$$2^{2x}+2^{x+2}-12=0$$
$$\left(2^x\right)^2+2^2\cdot 2^x-12=0$$
$$\left(2^x-2\right)\left(2^x+6\right)=0$$
$$2^x-2=0 \quad \text{or} \quad 2^x+6=0$$
$$2^x=2 \quad \text{or} \quad 2^x=-6$$
$$x=1 \qquad \text{No solution}$$
The solution set is $\{1\}$.

23.
$$2^x=10$$
$$\log\left(2^x\right)=\log 10$$
$$x\log 2=1$$
$$x=\frac{1}{\log 2}\approx 3.322$$
The solution set is $\left\{\frac{1}{\log 2}\approx 3.322\right\}$

24.
$$3^x=14$$
$$\log\left(3^x\right)=\log 14$$
$$x\log 3=\log 14$$
$$x=\frac{\log 14}{\log 3}\approx 2.402$$
The solution set is $\left\{\frac{\log 14}{\log 3}\approx 2.402\right\}$

25.
$$8^{-x}=1.2$$
$$\log\left(8^{-x}\right)=\log(1.2)$$
$$-x\log 8=\log(1.2)$$
$$x=\frac{\log(1.2)}{-\log 8}\approx -0.088$$
The solution set is $\left\{\frac{\log(1.2)}{-\log 8}\approx -0.088\right\}$.

26.
$$2^{-x}=1.5$$
$$\log\left(2^{-x}\right)=\log(1.5)$$
$$-x\log 2=\log(1.5)$$
$$x=\frac{\log(1.5)}{-\log 2}\approx -0.585$$
The solution set is $\left\{\frac{\log(1.5)}{-\log 2}\approx -0.585\right\}$.

27.
$$3^{1-2x}=4^x$$
$$\log\left(3^{1-2x}\right)=\log\left(4^x\right)$$
$$(1-2x)\log 3=x\log 4$$
$$\log 3-2x\log 3=x\log 4$$
$$\log 3=x\log 4+2x\log 3$$
$$\log 3=x(\log 4+2\log 3)$$
$$x=\frac{\log 3}{\log 4+2\log 3}\approx 0.307$$
The solution set is $\left\{\frac{\log 3}{\log 4+2\log 3}\approx 0.307\right\}$.

28.
$$2^{x+1}=5^{1-2x}$$
$$\log\left(2^{x+1}\right)=\log\left(5^{1-2x}\right)$$
$$(x+1)\log 2=(1-2x)\log 5$$
$$x\log 2+\log 2=\log 5-2x\log 5$$
$$x\log 2+2x\log 5=\log 5-\log 2$$
$$x(\log 2+2\log 5)=\log 5-\log 2$$
$$x=\frac{\log 5-\log 2}{\log 2+2\log 5}\approx 0.234$$
The solution set is $\left\{\frac{\log 5-\log 2}{\log 2+2\log 5}\approx 0.234\right\}$.

29.
$$\left(\frac{3}{5}\right)^x=7^{1-x}$$
$$\log\left(\frac{3}{5}\right)^x=\log\left(7^{1-x}\right)$$
$$x\log(3/5)=(1-x)\log 7$$
$$x\log(3/5)=\log 7-x\log 7$$
$$x\log(3/5)+x\log 7=\log 7$$
$$x\left(\log(3/5)+\log 7\right)=\log 7$$
$$x=\frac{\log 7}{\log(3/5)+\log 7}\approx 1.356$$
The solution set is $\left\{\frac{\log 7}{\log(3/5)+\log 7}\approx 1.356\right\}$.

30.
$$\left(\frac{4}{3}\right)^{1-x} = 5^x$$
$$\log\left(\frac{4}{3}\right)^{1-x} = \log\left(5^x\right)$$
$$(1-x)\log(4/3) = x\log 5$$
$$\log(4/3) - x\log(4/3) = x\log 5$$
$$\log(4/3) = x\log 5 + x\log(4/3)$$
$$\log(4/3) = x\left(\log 5 + \log(4/3)\right)$$
$$x = \frac{\log(4/3)}{\log 5 + \log(4/3)}$$
$$\approx 0.152$$
The solution set is $\left\{\frac{\log(4/3)}{\log 5 + \log(4/3)} \approx 0.152\right\}$.

31.
$$1.2^x = (0.5)^{-x}$$
$$\log 1.2^x = \log(0.5)^{-x}$$
$$x\log(1.2) = -x\log(0.5)$$
$$x\log(1.2) + x\log(0.5) = 0$$
$$x\left(\log(1.2) + \log(0.5)\right) = 0$$
$$x = 0$$
The solution set is $\{0\}$.

32.
$$0.3^{1+x} = 1.7^{2x-1}$$
$$\log\left(0.3^{1+x}\right) = \log\left(1.7^{2x-1}\right)$$
$$(1+x)\log(0.3) = (2x-1)\log(1.7)$$
$$\log(0.3) + x\log(0.3) = 2x\log(1.7) - \log(1.7)$$
$$x\log(0.3) - 2x\log(1.7) = -\log(1.7) - \log(0.3)$$
$$x\left(\log(0.3) - 2\log(1.7)\right) = -\log(1.7) - \log(0.3)$$
$$x = \frac{-\log(1.7) - \log(0.3)}{\log(0.3) - 2\log(1.7)} \approx -0.297$$
The solution set is
$\left\{\frac{-\log(1.7) - \log(0.3)}{\log(0.3) - 2\log(1.7)} \approx -0.297\right\}$.

33.
$$\pi^{1-x} = e^x$$
$$\ln \pi^{1-x} = \ln e^x$$
$$(1-x)\ln \pi = x$$
$$\ln \pi - x\ln \pi = x$$
$$\ln \pi = x + x\ln \pi$$
$$\ln \pi = x(1 + \ln \pi)$$
$$x = \frac{\ln \pi}{1 + \ln \pi} \approx 0.534$$
The solution set is $\left\{\frac{\ln \pi}{1 + \ln \pi} \approx 0.534\right\}$.

34.
$$e^{x+3} = \pi^x$$
$$\ln e^{x+3} = \ln \pi^x$$
$$x + 3 = x\ln \pi$$
$$x - x\ln \pi = -3$$
$$x(1 - \ln \pi) = -3$$
$$x = \frac{-3}{1 - \ln \pi} \approx 20.728$$
The solution set is $\left\{\frac{-3}{1 - \ln \pi} \approx 20.728\right\}$.

35.
$$5\left(2^{3x}\right) = 8$$
$$2^{3x} = \frac{8}{5}$$
$$\log 2^{3x} = \log\left(\frac{8}{5}\right)$$
$$3x\log 2 = \log(8/5)$$
$$x = \frac{\log(8/5)}{3\log 2} \approx 0.226$$
The solution set is $\left\{\frac{\log(8/5)}{3\log 2} \approx 0.226\right\}$.

36.
$$0.3\left(4^{0.2x}\right) = 0.2$$
$$4^{0.2x} = \frac{2}{3}$$
$$\log 4^{0.2x} = \log\left(\frac{2}{3}\right)$$
$$0.2x\log 4 = \log(2/3)$$
$$x = \frac{\log(2/3)}{0.2\log 4} \approx -1.462$$
The solution set is $\left\{\frac{\log(2/3)}{0.2\log 4} \approx -1.462\right\}$.

37. $16^x + 4^{x+1} - 3 = 0$

$\left(4^2\right)^x + 4 \cdot 4^x - 3 = 0$

$\left(4^x\right)^2 + 4 \cdot 4^x - 3 = 0$

Let $u = 4^x$.

$u^2 + 4u - 3 = 0$

$a = 1, b = 4, c = -3$

$$u = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} = \frac{-4 \pm \sqrt{28}}{2}$$

$$= \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$$

Therefore, we get

$\cancel{4^x = -2 - \sqrt{7}}$ or $4^x = -2 + \sqrt{7}$

$x = \log_4\left(-2 + \sqrt{7}\right)$

(we ignore the first solution since 4^x is never negative)

The solution set is $\left\{\log_4\left(-2 + \sqrt{7}\right)\right\} \approx \{-0.315\}$.

38. $9^x - 3^{x+1} + 1 = 0$

$\left(3^2\right)^x - 3 \cdot 3^x + 1 = 0$

$\left(3^x\right)^2 - 3 \cdot 3^x + 1 = 0$

Let $u = 3^x$.

$u^2 - 3u + 1 = 0$

$a = 1, b = -3, c = 1$

$$u = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2}$$

Therefore, we get

$3^x = \dfrac{3 \pm \sqrt{5}}{2}$

$x = \log_3\left(\dfrac{3 \pm \sqrt{5}}{2}\right)$

The solution set is

$\left\{\log_3\left(\dfrac{3 - \sqrt{5}}{2}\right), \log_3\left(\dfrac{3 + \sqrt{5}}{2}\right)\right\}$

$\approx \{-0.876, 0.876\}$.

39. $25^x - 8 \cdot 5^x = -16$

$\left(5^2\right)^x - 8 \cdot 5^x = -16$

$\left(5^x\right)^2 - 8 \cdot 5^x = -16$

Let $u = 5^x$.

$u^2 - 8u = -16$

$u^2 - 8u + 16 = 0$

$(u - 4)^2 = 0$

$u = 4$

Therefore, we get

$5^x = 4$

$x = \log_5 4$

The solution set is $\{\log_5 4\} \approx \{0.861\}$.

40. $36^x - 6 \cdot 6^x = -9$

$\left(6^2\right)^x - 6 \cdot 6^x + 9 = 0$

$\left(6^x\right)^2 - 6 \cdot 6^x + 9 = 0$

$\left(6^x - 3\right)^2 = 0$

$6^x = 3$

$x = \log_6 3$

The solution set is $\{\log_6 3\} \approx \{0.613\}$.

41. $3 \cdot 4^x + 14 \cdot 2^x + 8 = 0$

$3 \cdot \left(2^2\right)^x + 14 \cdot 2^x + 8 = 0$

$3 \cdot \left(2^x\right)^2 + 14 \cdot 2^x + 8 = 0$

Let $u = 2^x$.

$3u^2 + 14u + 8 = 0$

$(3u + 2)(u + 4) = 0$

$3u + 2 = 0$ or $u + 4 = 0$

$3u = -2$ $\quad u = -4$

$u = -\dfrac{2}{3}$

Therefore, we get

$2^x = -4$ or $2^x = -\dfrac{2}{3}$

Since $2^x > 0$ for all *x*, the equation has no real solution.

42. $2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$

$2 \cdot \left(7^2\right)^x + 11 \cdot 7^x + 5 = 0$

$2 \cdot \left(7^x\right)^2 + 11 \cdot 7^x + 5 = 0$

Let $u = 7^x$.

$2u^2 + 11u + 5 = 0$

$(2u+1)(u+5) = 0$

$2u + 1 = 0$ or $u + 5 = 0$

$2u = -1$ $\quad$ $u = -5$

$u = -\frac{1}{2}$

Therefore, we get

$7^x = -\frac{1}{2}$ or $7^x = -5$

Since $7^x > 0$ for all x, the equation has no real solution.

43. $4^x - 10 \cdot 4^{-x} = 3$

Multiply both sides of the equation by 4^x.

$\left(4^x\right)^2 - 10 \cdot 4^{-x} \cdot 4^x = 3 \cdot 4^x$

$\left(4^x\right)^2 - 10 = 3 \cdot 4^x$

$\left(4^x\right)^2 - 3 \cdot 4^x - 10 = 0$

$\left(4^x - 5\right)\left(4^x + 2\right) = 0$

$4^x - 5 = 0$ or $4^x + 2 = 0$

$4^x = 5$ $\quad$ $\cancel{4^x = -2}$

$x = \log_4 5$

The solution set is $\{\log_4 5\} \approx \{1.161\}$.

44. $3^x - 14 \cdot 3^{-x} = 5$

Multiply both sides of the equation by 3^x.

$\left(3^x\right)^2 - 14 \cdot 3^{-x} \cdot 3^x = 5 \cdot 3^x$

$\left(3^x\right)^2 - 14 = 5 \cdot 3^x$

$\left(3^x\right)^2 - 5 \cdot 3^x - 14 = 0$

$\left(3^x - 7\right)\left(3^x + 2\right) = 0$

$3^x - 7 = 0$ or $3^x + 2 = 0$

$3^x = 7$ $\quad$ $\cancel{3^x = -2}$

$x = \log_3 7$

The solution set is $\{\log_3 7\} \approx \{1.771\}$.

45. $\log_a(x-1) - \log_a(x+6) = \log_a(x-2) - \log_a(x+3)$

$$\log_a\left(\frac{x-1}{x+6}\right) = \log_a\left(\frac{x-2}{x+3}\right)$$

$$\frac{x-1}{x+6} = \frac{x-2}{x+3}$$

$$(x-1)(x+3) = (x-2)(x+6)$$

$$x^2 + 2x - 3 = x^2 + 4x - 12$$

$$2x - 3 = 4x - 12$$

$$9 = 2x$$

$$x = \frac{9}{2}$$

Since each of the original logarithms is defined for $x = \frac{9}{2}$, the solution set is $\left\{\frac{9}{2}\right\}$.

46. $\log_a x + \log_a(x-2) = \log_a(x+4)$

$\log_a\left(x(x-2)\right) = \log_a(x+4)$

$x(x-2) = x+4$

$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or $x = -1$

Since $\log_a(-1)$ is undefined, the solution set is $\{4\}$.

47. $\log_{1/3}(x^2 + x) - \log_{1/3}(x^2 - x) = -1$

$$\log_{1/3}\left(\frac{x^2+x}{x^2-x}\right) = -1$$

$$\frac{x^2+x}{x^2-x} = \left(\frac{1}{3}\right)^{-1}$$

$$\frac{x^2+x}{x^2-x} = 3$$

$$x^2 + x = 3\left(x^2 - x\right)$$

$$x^2 + x = 3x^2 - 3x$$

$$-2x^2 + 4x = 0$$

$$-2x(x-2) = 0$$

$-2x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

Since each of the original logarithms are not defined for $x = 0$, but are defined for $x = 2$, the solution set is $\{2\}$.

48. $\log_4(x^2-9)-\log_4(x+3)=3$

$$\log_4\left(\frac{x^2-9}{x+3}\right)=3$$
$$\frac{(x-3)(x+3)}{x+3}=4^3$$
$$x-3=64$$
$$x=67$$

Since each of the original logarithms is defined for $x=67$, the solution set is $\{67\}$.

49. $\log_2(x+1)-\log_4 x=1$

$$\log_2(x+1)-\frac{\log_2 x}{\log_2 4}=1$$
$$\log_2(x+1)-\frac{\log_2 x}{2}=1$$
$$2\log_2(x+1)-\log_2 x=2$$
$$\log_2(x+1)^2-\log_2 x=2$$
$$\log_2\left(\frac{(x+1)^2}{x}\right)=2$$
$$\frac{(x+1)^2}{x}=2^2$$
$$x^2+2x+1=4x$$
$$x^2-2x+1=0$$
$$(x-1)^2=0$$
$$x-1=0$$
$$x=1$$

Since each of the original logarithms is defined for $x=1$, the solution set is $\{1\}$.

50. $\log_2(3x+2)-\log_4 x=3$

$$\log_2(3x+2)-\frac{\log_2 x}{\log_2 4}=3$$
$$\log_2(3x+2)-\frac{\log_2 x}{2}=3$$
$$2\log_2(3x+2)-\log_2 x=6$$
$$\log_2(3x+2)^2-\log_2 x=6$$
$$\log_2\left(\frac{(3x+2)^2}{x}\right)=6$$
$$\frac{(3x+2)^2}{x}=2^6$$
$$9x^2+12x+4=64x$$
$$9x^2-52x+4=0$$

$$x=\frac{52\pm\sqrt{(-52)^2-4(9)(4)}}{2(9)}$$
$$=\frac{52\pm\sqrt{2560}}{18}$$
$$=\frac{52\pm16\sqrt{10}}{18}$$
$$=\frac{26\pm8\sqrt{10}}{9}$$
$$\approx 5.700 \text{ or } 0.078$$

Since each of the original logarithms is defined for $x=0.078$ and $x=5.700$, the solution set is $\left\{\frac{26-8\sqrt{10}}{9},\ \frac{26+8\sqrt{10}}{9}\right\}\approx\{0.078,\ 5.700\}$.

51. $\log_{16}x+\log_4 x+\log_2 x=7$

$$\frac{\log_2 x}{\log_2 16}+\frac{\log_2 x}{\log_2 4}+\log_2 x=7$$
$$\frac{\log_2 x}{4}+\frac{\log_2 x}{2}+\log_2 x=7$$
$$\log_2 x+2\log_2 x+4\log_2 x=28$$
$$7\log_2 x=28$$
$$\log_2 x=4$$
$$x=2^4=16$$

Since each of the original logarithms is defined for $x=16$, the solution set is $\{16\}$.

52. $\log_9 x+3\log_3 x=14$

$$\frac{\log_3 x}{\log_3 9}+3\log_3 x=14$$
$$\frac{\log_3 x}{2}+3\log_3 x=14$$
$$\frac{7}{2}\log_3 x=14$$
$$\log_3 x=4$$
$$x=3^4=81$$

Since each of the original logarithms is defined for $x=81$, the solution set is $\{81\}$.

53. $\left(\sqrt[3]{2}\right)^{2-x} = 2^{x^2}$

$\left(2^{1/3}\right)^{2-x} = 2^{x^2}$

$2^{\frac{1}{3}(2-x)} = 2^{x^2}$

$\frac{1}{3}(2-x) = x^2$

$2-x = 3x^2$

$3x^2 + x - 2 = 0$

$(3x-2)(x+1) = 0$

$x = \frac{2}{3}$ or $x = -1$

The solution set is $\left\{-1, \frac{2}{3}\right\}$.

54. $\log_2 x^{\log_2 x} = 4$

$\log_2 x \cdot \log_2 x = 4$

$(\log_2 x)^2 = 4$

$\log_2 x = -2$ or $\log_2 x = 2$

$x = 2^{-2}$ or $x = 2^2$

$x = \frac{1}{4}$ or $x = 4$

Since each of the original logarithms is defined for both $x = \frac{1}{4}$ and $x = 4$, the solution set is $\left\{\frac{1}{4}, 4\right\}$.

55. $\frac{e^x + e^{-x}}{2} = 1$

$e^x + e^{-x} = 2$

$e^x\left(e^x + e^{-x}\right) = 2e^x$

$e^{2x} + 1 = 2e^x$

$(e^x)^2 - 2e^x + 1 = 0$

$\left(e^x - 1\right)^2 = 0$

$e^x - 1 = 0$

$e^x = 1$

$x = 0$

The solution set is $\{0\}$.

56. $\frac{e^x + e^{-x}}{2} = 3$

$e^x + e^{-x} = 6$

$e^x\left(e^x + e^{-x}\right) = 6e^x$

$e^{2x} + 1 = 6e^x$

$(e^x)^2 - 6e^x + 1 = 0$

$e^x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$

$= \frac{6 \pm \sqrt{32}}{2}$

$= \frac{6 \pm 4\sqrt{2}}{2}$

$= 3 \pm 2\sqrt{2}$

$x = \ln\left(3 - 2\sqrt{2}\right)$ or $x = \ln\left(3 + 2\sqrt{2}\right)$

$x \approx -1.763$ or $x \approx 1.763$

The solution set is

$\left\{\ln\left(3 - 2\sqrt{2}\right),\ \ln\left(3 + 2\sqrt{2}\right)\right\} \approx \{-1.763, 1.763\}$.

57. $\frac{e^x - e^{-x}}{2} = 2$

$e^x - e^{-x} = 4$

$e^x\left(e^x - e^{-x}\right) = 4e^x$

$e^{2x} - 1 = 4e^x$

$(e^x)^2 - 4e^x - 1 = 0$

$e^x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$

$= \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2}$

$= 2 \pm \sqrt{5}$

$x = \ln\left(2 - \sqrt{5}\right)$ or $x = \ln\left(2 + \sqrt{5}\right)$

$x \approx \ln(-0.236)$ or $x \approx 1.444$

Since $\ln(-0.236)$ is undefined, the solution set is $\left\{\ln\left(2 + \sqrt{5}\right) \approx 1.444\right\}$.

58.
$$\frac{e^x - e^{-x}}{2} = -2$$
$$e^x - e^{-x} = -4$$
$$e^x\left(e^x - e^{-x}\right) = -4e^x$$
$$e^{2x} - 1 = -4e^x$$
$$(e^x)^2 + 4e^x - 1 = 0$$
$$e^x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{20}}{2}$$
$$= \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$
$$x = \ln\left(-2 - \sqrt{5}\right) \text{ or } x = \ln\left(-2 + \sqrt{5}\right)$$
$$x \approx \ln\left(-4.236\right) \text{ or } x \approx -1.444$$

Since $\ln\left(-4.236\right)$ is undefined, the solution set is $\left\{\ln\left(-2 + \sqrt{5}\right) \approx -1.444\right\}$.

59.
$$\log_5 x + \log_3 x = 1$$
$$\frac{\log x}{\log 5} + \frac{\log x}{\log 3} = 1$$
$$\left(\log x\right)\left(\frac{1}{\log 5} + \frac{1}{\log 3}\right) = 1$$
$$\log x = \frac{1}{\frac{1}{\log 5} + \frac{1}{\log 3}}$$
$$x = 10^{\left(\frac{1}{\frac{1}{\log 5} + \frac{1}{\log 3}}\right)} \approx 1.921$$

The solution set is $\left\{10^{\frac{1}{\frac{1}{\log 5} + \frac{1}{\log 3}}} \approx 1.921\right\}$.

60.
$$\log_2 x + \log_6 x = 3$$
$$\frac{\log x}{\log 2} + \frac{\log x}{\log 6} = 3$$
$$\left(\log x\right)\left(\frac{1}{\log 2} + \frac{1}{\log 6}\right) = 3$$
$$\log x = \frac{3}{\frac{1}{\log 2} + \frac{1}{\log 6}}$$
$$x = 10^{\left(\frac{3}{\frac{1}{\log 2} + \frac{1}{\log 6}}\right)} \approx 4.479$$

The solution set is $\left\{10^{\left(\frac{3}{\frac{1}{\log 2} + \frac{1}{\log 6}}\right)} \approx 4.479\right\}$.

61. a.
$$f\left(x\right) = 3$$
$$\log_2\left(x + 3\right) = 3$$
$$x + 3 = 2^3$$
$$x + 3 = 8$$
$$x = 5$$

The point $\left(5, 3\right)$ is on the graph of *f*.

b.
$$g\left(x\right) = 4$$
$$\log_2\left(3x + 1\right) = 4$$
$$3x + 1 = 2^4$$
$$3x + 1 = 16$$
$$3x = 15$$
$$x = 5$$

The point $\left(5, 4\right)$ is on the graph of *g*.

c.
$$f\left(x\right) = g\left(x\right)$$
$$\log_2\left(x + 3\right) = \log_2\left(3x + 1\right)$$
$$x + 3 = 3x + 1$$
$$2 = 2x$$
$$1 = x$$

The graphs intersect when $x = 1$. That is, at the point $\left(1, 2\right)$.

d.
$$\begin{aligned} f(x)+g(x)&=7\\ \log_2(x+3)+\log_2(3x+1)&=7\\ \log_2\left[(x+3)(3x+1)\right]&=7\\ (x+3)(3x+1)&=2^7\\ 3x^2+10x+3&=128\\ 3x^2+10x-125&=0\\ (3x+25)(x-5)&=0 \end{aligned}$$
$$\cancel{3x+25=0 \quad 3x=-25 \quad x=-\frac{25}{3}} \quad \text{or} \quad x-5=0 \quad x=5$$
The solution set is $\{5\}$.

e.
$$\begin{aligned} f(x)-g(x)&=2\\ \log_2(x+3)-\log_2(3x+1)&=2\\ \log_2\frac{x+3}{3x+1}&=2\\ \frac{x+3}{3x+1}&=2^2\\ x+3&=4(3x+1)\\ x+3&=12x+4\\ -1&=11x\\ -\frac{1}{11}&=x \end{aligned}$$
The solution set is $\left\{-\frac{1}{11}\right\}$.

62. a.
$$\begin{aligned} f(x)&=2\\ \log_3(x+5)&=2\\ x+5&=3^2\\ x+5&=9\\ x&=4 \end{aligned}$$
The point $(4,2)$ is on the graph of *f*.

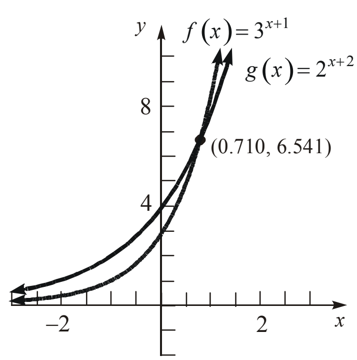
b.
$$\begin{aligned} g(x)&=3\\ \log_3(x-1)&=3\\ x-1&=3^3\\ x-1&=27\\ x&=28 \end{aligned}$$
The point $(28,3)$ is on the graph of *g*.

c.
$$\begin{aligned} f(x)&=g(x)\\ \log_3(x+5)&=\log_3(x-1)\\ x+5&=x-1\\ 5&=-1 \quad \text{False} \end{aligned}$$
This is a contradiction. Therefore, the graphs do not intersect.

d.
$$\begin{aligned} f(x)+g(x)&=3\\ \log_3(x+5)+\log_3(x-1)&=3\\ \log_3\left[(x+5)(x-1)\right]&=3\\ (x+5)(x-1)&=3^3\\ x^2+4x-5&=27\\ x^2+4x-32&=0\\ (x+8)(x-4)&=0 \end{aligned}$$
$$\cancel{x+8=0 \quad x=-8} \quad \text{or} \quad x-4=0 \quad x=4$$
The solution set is $\{4\}$.

e.
$$\begin{aligned} f(x)-g(x)&=2\\ \log_3(x+5)-\log_3(x-1)&=2\\ \log_3\frac{x+5}{x-1}&=2\\ \frac{x+5}{x-1}&=3^2\\ x+5&=9(x-1)\\ x+5&=9x-9\\ 14&=8x\\ \frac{7}{4}&=x \end{aligned}$$
The solution set is $\left\{\frac{7}{4}\right\}$.

63. a.

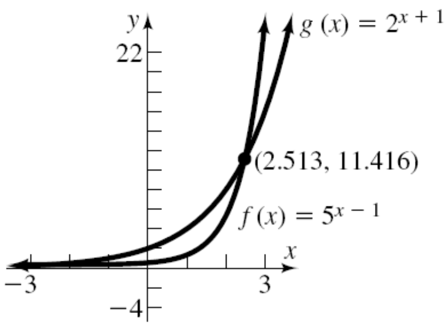


b.
$$f(x)=g(x)$$
$$3^{x+1}=2^{x+2}$$
$$\ln\left(3^{x+1}\right)=\ln\left(2^{x+2}\right)$$
$$(x+1)\ln 3=(x+2)\ln 2$$
$$x\ln 3+\ln 3=x\ln 2+2\ln 2$$
$$x\ln 3-x\ln 2=2\ln 2-\ln 3$$
$$x(\ln 3-\ln 2)=2\ln 2-\ln 3$$
$$x=\frac{2\ln 2-\ln 3}{\ln 3-\ln 2}\approx 0.710$$
$$f\left(\frac{2\ln 2-\ln 3}{\ln 3-\ln 2}\right)\approx 6.541$$

The intersection point is roughly $(0.710, 6.541)$.

c. Based on the graph, $f(x)>g(x)$ for $x>0.710$. The solution set is $\{x \mid x>0.710\}$ or $(0.710,\infty)$.

64. a.

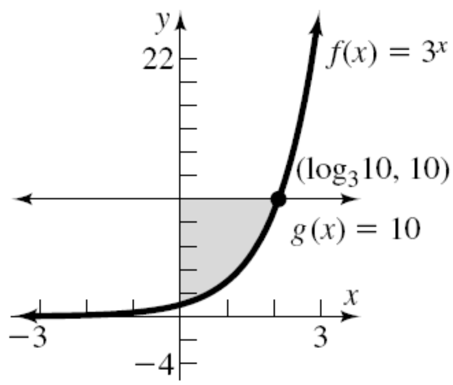


b.
$$f(x)=g(x)$$
$$5^{x-1}=2^{x+1}$$
$$\ln\left(5^{x-1}\right)=\ln\left(2^{x+1}\right)$$
$$(x-1)\ln 5=(x+1)\ln 2$$
$$x\ln 5-\ln 5=x\ln 2+\ln 2$$
$$x\ln 5-x\ln 2=\ln 5+\ln 2$$
$$x(\ln 5-\ln 2)=\ln 5+\ln 2$$
$$x=\frac{\ln 5+\ln 2}{\ln 5-\ln 2}\approx 2.513$$
$$f\left(\frac{\ln 5+\ln 2}{\ln 5-\ln 2}\right)\approx 11.416$$

The intersection point is roughly $(2.513, 11.416)$.

c. Based on the graph, $f(x)>g(x)$ for $x>2.513$. The solution set is $\{x \mid x>2.513\}$ or $(2.513,\infty)$.

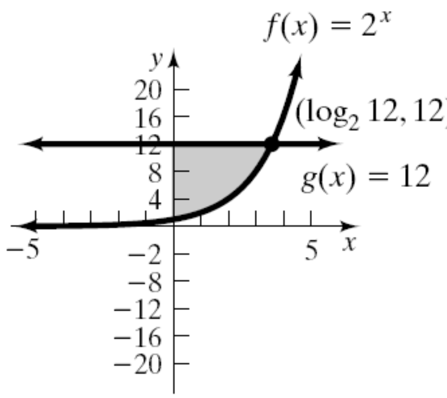
65. a., b.



c.
$$f(x)=g(x)$$
$$3^x=10$$
$$x=\log_3 10$$

The intersection point is $(\log_3 10, 10)$.

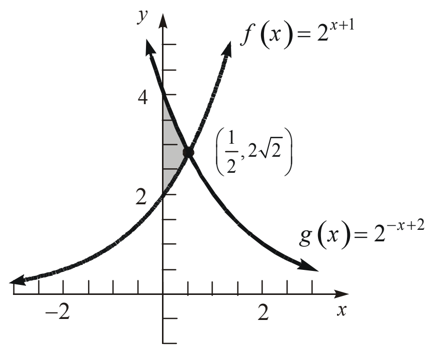
66. a., b.



c.
$$f(x)=g(x)$$
$$2^x=12$$
$$x=\log_2 12$$

The intersection point is $(\log_2 12, 12)$.

67. a., b.

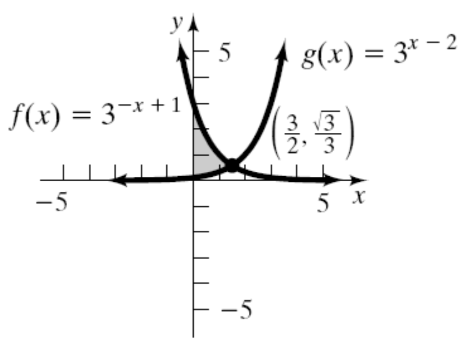


c. $f(x)=g(x)$

$$2^{x+1}=2^{-x+2}$$
$$x+1=-x+2$$
$$2x=1$$
$$x=\frac{1}{2}$$
$$f\left(\frac{1}{2}\right)=2^{1/2+1}=2^{3/2}=2\sqrt{2}$$

The intersection point is $\left(\frac{1}{2},2\sqrt{2}\right)$.

68. a., b.



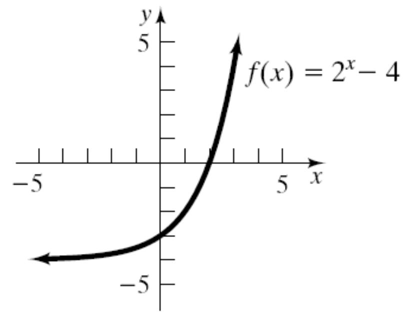
c. $f(x)=g(x)$

$$3^{-x+1}=3^{x-2}$$
$$-x+1=x-2$$
$$-2x=-3$$
$$x=\frac{3}{2}$$
$$f\left(\frac{3}{2}\right)=3^{-3/2+1}=3^{-1/2}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

The intersection point is $\left(\frac{3}{2},\frac{\sqrt{3}}{3}\right)$.

69. a. $f(x)=2^x-4$

Using the graph of $y=2^x$, shift the graph down 4 units.



b. $f(x)=0$

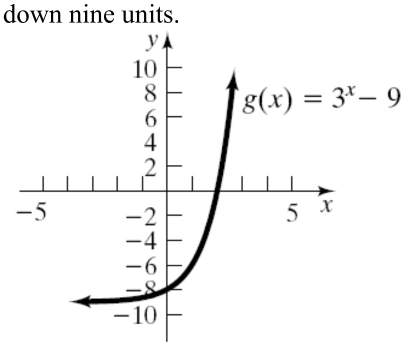
$$2^x-4=0$$
$$2^x=4$$
$$2^x=2^2$$
$$x=2$$

The zero of *f* is 2.

c. Based on the graph, $f(x)<0$ when $x<2$. The solution set is $\{x\,|\,x<2\}$ or $(-\infty,2)$.

70. a. $g(x)=3^x-9$

Using the graph of $y=3^x$, shift the graph down nine units.



b. $g(x)=0$

$$3^x-9=0$$
$$3^x=9$$
$$3^x=3^2$$
$$x=2$$

The zero of *g* is 2.

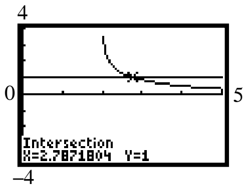
c. Based on the graph, $g(x)>0$ when $x>2$. The solution set is $\{x\,|\,x>2\}$ or $(2,\infty)$.

71. $\log_5(x+1)-\log_4(x-2)=1$

Using INTERSECT to solve:

$y_1=\ln(x+1)/\ln(5)-\ln(x-2)/\ln(4)$

$y_2=1$



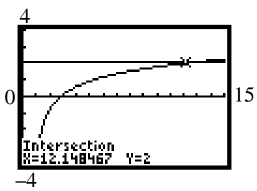
Thus, $x\approx 2.79$, so the solution set is $\{2.79\}$.

72. $\log_2(x-1)-\log_6(x+2)=2$

Using INTERSECT to solve:

$y_1=\ln(x-1)/\ln(2)-\ln(x+2)/\ln(6)$

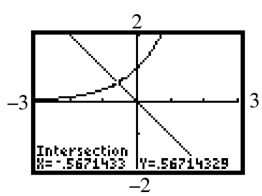
$y_2=2$



Thus, $x\approx 12.15$, so the solution set is $\{12.15\}$.

73. $e^x=-x$

Using INTERSECT to solve: $y_1=e^x$; $y_2=-x$

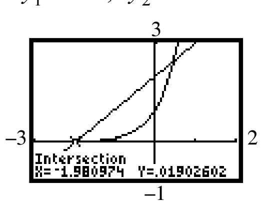


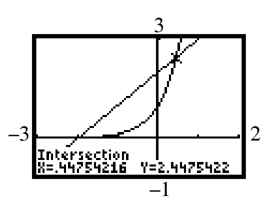
Thus, $x\approx -0.57$, so the solution set is $\{-0.57\}$.

74. $e^{2x}=x+2$

Using INTERSECT to solve:

$y_1=e^{2x}$; $y_2=x+2$



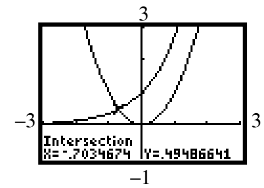


Thus, $x\approx -1.98$ or $x\approx 0.45$, so the solution set is $\{-1.98,\ 0.45\}$.

75. $e^x=x^2$

Using INTERSECT to solve:

$y_1=e^x$; $y_2=x^2$

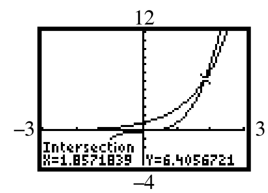


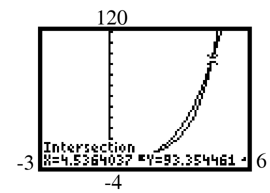
Thus, $x\approx -0.70$, so the solution set is $\{-0.70\}$.

76. $e^x=x^3$

Using INTERSECT to solve:

$y_1=e^x$; $y_2=x^3$



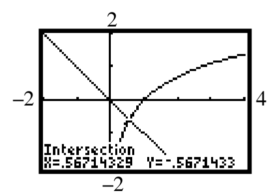


Thus, $x\approx 1.86$ or $x\approx 4.54$, so the solution set is $\{1.86,\ 4.54\}$.

77. $\ln x=-x$

Using INTERSECT to solve:

$y_1=\ln x$; $y_2=-x$

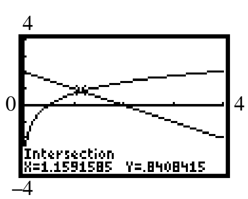


Thus, $x\approx 0.57$, so the solution set is $\{0.57\}$.

78. $\ln(2x)=-x+2$

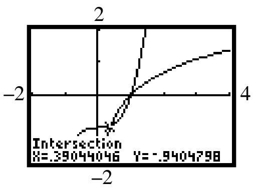
Using INTERSECT to solve:

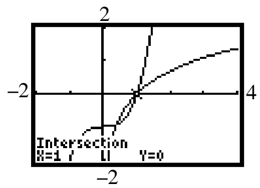
$y_1=\ln(2x)$; $y_2=-x+2$



Thus, $x\approx 1.16$, so the solution set is $\{1.16\}$.

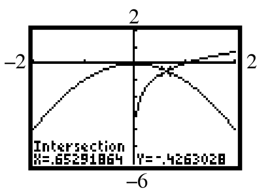
79. $\ln x = x^3 - 1$
Using INTERSECT to solve:
$y_1 = \ln x;\ \ y_2 = x^3 - 1$





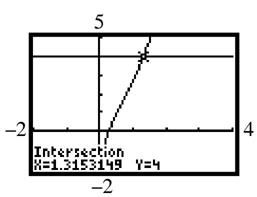
Thus, $x \approx 0.39$ or $x = 1$, so the solution set is $\{0.39,\ 1\}$.

80. $\ln x = -x^2$
Using INTERSECT to solve:
$y_1 = \ln x;\ \ y_2 = -x^2$



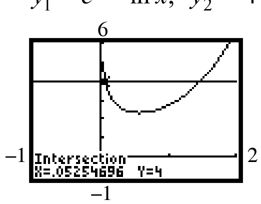
Thus, $x \approx 0.65$, so the solution set is $\{0.65\}$.

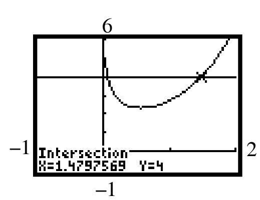
81. $e^x + \ln x = 4$
Using INTERSECT to solve:
$y_1 = e^x + \ln x;\ \ y_2 = 4$



Thus, $x \approx 1.32$, so the solution set is $\{1.32\}$.

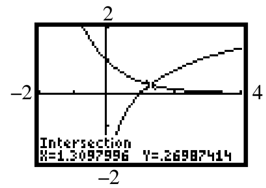
82. $e^x - \ln x = 4$
Using INTERSECT to solve:
$y_1 = e^x - \ln x;\ \ y_2 = 4$





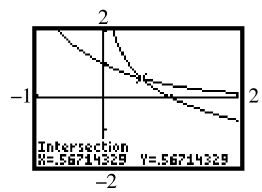
Thus, $x \approx 0.05$ or $x \approx 1.48$, so the solution set is $\{0.05,\ 1.48\}$.

83. $e^{-x} = \ln x$
Using INTERSECT to solve:
$y_1 = e^{-x};\ \ y_2 = \ln x$



Thus, $x \approx 1.31$, so the solution set is $\{1.31\}$.

84. $e^{-x} = -\ln x$
Using INTERSECT to solve:
$y_1 = e^{-x};\ \ y_2 = -\ln x$



Thus, $x \approx 0.57$, so the solution set is $\{0.57\}$

85. a.

$$
\begin{aligned}
282(1.011)^{t-2000} &= 303 \\
(1.011)^{t-2000} &= \frac{303}{282} \\
\log(1.011)^{t-2000} &= \log\left(\frac{101}{94}\right) \\
(t-2000)\log(1.011) &= \log\left(\frac{101}{94}\right) \\
t - 2000 &= \frac{\log(101/94)}{\log(1.011)} \\
t &= \frac{\log(101/94)}{\log(1.011)} + 2000 \\
&\approx 2006.57
\end{aligned}
$$

According to the model, the population of the U.S. will reach 303 million people around the middle of the year 2006.

b.
$$\begin{aligned}282(1.011)^{t-2000} &= 355\\ (1.011)^{t-2000} &= \frac{355}{282}\\ \log(1.011)^{t-2000} &= \log\left(\frac{355}{282}\right)\\ (t-2000)\log(1.011) &= \log\left(\frac{355}{282}\right)\\ t-2000 &= \frac{\log(355/282)}{\log(1.011)}\\ t &= \frac{\log(355/282)}{\log(1.011)}+2000\\ &\approx 2021.04\end{aligned}$$

According to the model, the population of the U.S. will reach 355 million people in the beginning of the year 2021.

86. a.
$$\begin{aligned}6.08(1.026)^{t-2000} &= 9.17\\ (1.026)^{t-2000} &= \frac{9.17}{6.08}\\ \log(1.026)^{t-2000} &= \log\left(\frac{9.17}{6.08}\right)\\ (t-2000)\log(1.026) &= \log(9.17/6.08)\\ t-2000 &= \frac{\log(9.17/6.08)}{\log(1.026)}\\ t &= \frac{\log(9.17/6.08)}{\log(1.026)}+2000\\ &\approx 2016.01\end{aligned}$$

According to the model, the population of the world will reach 6.08 billion people in the beginning of the year 2016.

b.
$$\begin{aligned}6.08(1.026)^{t-2000} &= 11.55\\ (1.026)^{t-2000} &= \frac{11.55}{6.08}\\ \log(1.026)^{t-2000} &= \log\left(\frac{11.55}{6.08}\right)\\ (t-2000)\log(1.026) &= \log(11.55/6.08)\\ t-2000 &= \frac{\log(11.55/6.08)}{\log(1.026)}\\ t &= \frac{\log(11.55/6.08)}{\log(1.026)}+2000\\ &\approx 2025.00\end{aligned}$$

According to the model, the population of the world will reach 11.55 billion people in the beginning of the year 2025.

87. a.
$$\begin{aligned}14{,}512(0.82)^{t} &= 9{,}000\\ (0.82)^{t} &= \frac{9{,}000}{14{,}512}\\ \log(0.82)^{t} &= \log\left(\frac{9{,}000}{14{,}512}\right)\\ t\log(0.82) &= \log\left(\frac{9{,}000}{14{,}512}\right)\\ t &= \frac{\log(9{,}000/14{,}512)}{\log(0.82)}\\ &\approx 2.4\end{aligned}$$

According to the model, the car will be worth \$9,000 after about 2.4 years.

b.
$$\begin{aligned}14{,}512(0.82)^{t} &= 4{,}000\\ (0.82)^{t} &= \frac{4{,}000}{14{,}512}\\ \log(0.82)^{t} &= \log\left(\frac{4{,}000}{14{,}512}\right)\\ t\log(0.82) &= \log\left(\frac{4{,}000}{14{,}512}\right)\\ t &= \frac{\log(4{,}000/14{,}512)}{\log(0.82)}\\ &\approx 6.5\end{aligned}$$

According to the model, the car will be worth \$4,000 after about 6.5 years.

c.
$$\begin{aligned}14{,}512(0.82)^{t} &= 2{,}000\\ (0.82)^{t} &= \frac{2{,}000}{14{,}512}\\ \log(0.82)^{t} &= \log\left(\frac{2{,}000}{14{,}512}\right)\\ t\log(0.82) &= \log\left(\frac{2{,}000}{14{,}512}\right)\\ t &= \frac{\log(2{,}000/14{,}512)}{\log(0.82)}\\ &\approx 10.0\end{aligned}$$

According to the model, the car will be worth \$2,000 after about 10 years.

88. a. $19,282(0.84)^t = 15,000$

$$(0.84)^t = \frac{15,000}{19,282}$$

$$\log(0.84)^t = \log\left(\frac{15,000}{19,282}\right)$$

$$t\log(0.84) = \log\left(\frac{15,000}{19,282}\right)$$

$$t = \frac{\log(15,000/19,282)}{\log(0.84)}$$

$$\approx 1.4$$

According to the model, the car will be worth \$15,000 after about 1.4 years.

b. $19,282(0.84)^t = 8,000$

$$(0.84)^t = \frac{8,000}{19,282}$$

$$\log(0.84)^t = \log\left(\frac{8,000}{19,282}\right)$$

$$t\log(0.84) = \log\left(\frac{8,000}{19,282}\right)$$

$$t = \frac{\log(8,000/19,282)}{\log(0.84)}$$

$$\approx 5.0$$

According to the model, the car will be worth \$8,000 after about 5 years.

c. $19,282(0.84)^t = 2,000$

$$(0.84)^t = \frac{2,000}{19,282}$$

$$\log(0.84)^t = \log\left(\frac{2,000}{19,282}\right)$$

$$t\log(0.84) = \log\left(\frac{2,000}{19,282}\right)$$

$$t = \frac{\log(2,000/19,282)}{\log(0.84)}$$

$$\approx 13.0$$

According to the model, the car will be worth \$2,000 after about 13 years.

89. Solution A: change to exponential expression; square root method; meaning of $\pm$; solve.

Solution B: $\log_a M^r = r\log_a M$; divide by 2; change to exponential expression; solve.

The power rule $\log_a M^r = r\log_a M$ only applies when $M > 0$. In this equation, $M = x-1$. Now, $x = -2$ causes $M = -2-1 = -3$. Thus, if we use the power rule, we lose the valid solution $x = -2$.

Section 4.7

1. $P = \$500,\ r = 0.06,\ t = 6 \text{ months} = 0.5 \text{ year}$

$I = Prt = (500)(0.06)(0.5) = \$15.00$

2. $P = \$5000,\ t = 9 \text{ months} = 0.75 \text{ year},\ I = \$500$

$$500 = 5000r(0.75)$$

$$\frac{500}{(5000)(0.75)} = r$$

$$0.1333 \approx r$$

The per annum interest rate was approximately 13.33%.

3. $P = \$100,\ r = 0.04,\ n = 4,\ t = 2$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 100\left(1+\frac{0.04}{4}\right)^{(4)(2)} \approx \$108.29$$

4. $P = \$50,\ r = 0.06,\ n = 12,\ t = 3$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 50\left(1+\frac{0.06}{12}\right)^{(12)(3)} \approx \$59.83$$

5. $P = \$500,\ r = 0.08,\ n = 4,\ t = 2.5$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 500\left(1+\frac{0.08}{4}\right)^{(4)(2.5)} \approx \$609.50$$

6. $P = \$300,\ r = 0.12,\ n = 12,\ t = 1.5$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 300\left(1+\frac{0.12}{12}\right)^{(12)(1.5)} \approx \$358.84$$

7. $P = \$600,\ r = 0.05,\ n = 365,\ t = 3$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 600\left(1+\frac{0.05}{365}\right)^{(365)(3)} \approx \$697.09$$

8. $P = \$700,\ r = 0.06,\ n = 365,\ t = 2$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 700\left(1+\frac{0.06}{365}\right)^{(365)(2)} \approx \$789.24$$

9. $P = \$10,\ r = 0.11,\ t = 2$

$A = Pe^{rt} = 10e^{(0.11)(2)} \approx \$12.46$

10. $P = \$40,\ r = 0.07,\ t = 3$

$A = Pe^{rt} = 40e^{(0.07)(3)} \approx \$49.35$

11. $P = \$100,\ r = 0.10,\ t = 2.25$

$A = Pe^{rt} = 100e^{(0.10)(2.25)} \approx \$125.23$

12. $P = \$100,\ r = 0.12,\ t = 3.75$

$A = Pe^{rt} = 100e^{(0.12)(3.75)} \approx \$156.83$

13. $A = \$100,\ r = 0.06,\ n = 12,\ t = 2$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 100\left(1+\frac{0.06}{12}\right)^{(-12)(2)} \approx \$88.72$$

14. $A = \$75,\ r = 0.08,\ n = 4,\ t = 3$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 75\left(1+\frac{0.08}{4}\right)^{(-4)(3)} \approx \$59.14$$

15. $A = \$1000,\ r = 0.06,\ n = 365,\ t = 2.5$

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$= 1000\left(1+\frac{0.06}{365}\right)^{(-365)(2.5)}$$
$$\approx \$860.72$$

16. $A = \$800,\ r = 0.07,\ n = 12,\ t = 3.5$

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$= 800\left(1+\frac{0.07}{12}\right)^{(-12)(3.5)}$$
$$\approx \$626.61$$

17. $A = \$600,\ r = 0.04,\ n = 4,\ t = 2$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 600\left(1+\frac{0.04}{4}\right)^{(-4)(2)} \approx \$554.09$$

18. $A = \$300,\ r = 0.03,\ n = 365,\ t = 4$

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$= 300\left(1+\frac{0.03}{365}\right)^{(-365)(4)}$$
$$\approx \$266.08$$

19. $A = \$80,\ r = 0.09,\ t = 3.25$

$P = Ae^{-rt} = 80e^{(-0.09)(3.25)} \approx \$59.71$

20. $A = \$800,\ r = 0.08,\ t = 2.5$

$P = Ae^{-rt} = 800e^{(-0.08)(2.5)} \approx \$654.98$

21. $A = \$400,\ r = 0.10,\ t = 1$

$P = Ae^{-rt} = 400e^{(-0.10)(1)} \approx \$361.93$

22. $A = \$1000,\ r = 0.12,\ t = 1$

$P = Ae^{-rt} = 1000e^{(-0.12)(1)} \approx \$886.92$

23. \$1000 invested for 1 year at $5\frac{1}{4}\%$

Compounded quarterly yields:

$$100\left(1+\frac{0.0525}{4}\right)^{(4)(1)} = \$1053.54\ .$$

The interest earned is

$\$1053.54 - \$1000.00 = \$53.54$

Thus, $I = Prt$

$$5.354 = 1000 \cdot r \cdot 1$$
$$r = \frac{5.354}{1000} = .05354$$

The effective interest rate is 5.354%.

24. Since the effective interest rate is 7%, we have:

$I = Prt$

$I = P \cdot 0.07 \cdot 1$

$I = 0.07P$

Thus, the amount in the account is

$A = P + 0.07P = 1.07P$

Let x be the required interest rate. Then,

$$1.07P = P\left(1+\frac{x}{4}\right)^{(4)(1)}$$

$$1.07 = \left(1+\frac{x}{4}\right)^{4}$$

$$\sqrt[4]{1.07} = 1+\frac{x}{4}$$

$$\sqrt[4]{1.07}-1 = \frac{x}{4}$$

$$x = 4\left(\sqrt[4]{1.07}-1\right)$$

$$x \approx 0.0682$$

Thus, an interest rate of 6.82% compounded quarterly has an effective interest rate of 7%.

25. $2P = P(1+r)^3$

$$2 = (1+r)^3$$

$$\sqrt[3]{2} = 1+r$$

$$r = \sqrt[3]{2}-1$$

$$\approx 1.26-1$$

$$= 0.26$$

$$r \approx 26\%$$

26. $2P = P(1+r)^{10}$

$$2 = (1+r)^{10}$$

$$\sqrt[10]{2} = 1+r$$

$$r = \sqrt[10]{2}-1$$

$$\approx 1.0718-1$$

$$= 0.0718$$

$$r \approx 7.18\%$$

27. 6% compounded quarterly:

$$A = 10,000\left(1+\frac{0.06}{4}\right)^{(4)(1)} = \$10,613.64$$

$6\frac{1}{4}\%$ compounded annually:

$$A = 10,000(1+0.0625)^1 = \$10,625$$

$6\frac{1}{4}\%$ compounded annually yields the larger amount.

28. 9% compounded quarterly:

$$A = 10,000\left(1+\frac{0.09}{4}\right)^{(4)(1)} \approx \$10,930.83$$

$9\frac{1}{4}\%$ compounded annually:

$$A = 10,000(1+0.0925)^1 = \$10,925$$

9% compounded quarterly yields the larger amount.

29. 9% compounded monthly:

$$A = 10,000\left(1+\frac{0.09}{12}\right)^{(12)(1)} = \$10,938.07$$

8.8% compounded daily:

$$A = 10,000\left(1+\frac{0.088}{365}\right)^{365} = \$10,919.77$$

9% compounded monthly yields the larger amount.

30. 8% compounded semiannually:

$$A = 10,000\left(1+\frac{0.08}{2}\right)^{(2)(1)} = \$10,816$$

7.9% compounded daily:

$$A = 10,000\left(1+\frac{0.079}{365}\right)^{365} = \$10,821.95$$

7.9% compounded daily yields the larger amount.

31. $2P = P\left(1+\frac{0.08}{12}\right)^{12t}$

$$2 \approx (1.006667)^{12t}$$

$$\ln 2 \approx 12t\ \ln(1.006667)$$

$$t \approx \frac{\ln 2}{12\ \ln(1.006667)} \approx 8.69$$

Compounded monthly, it will take about 8.69 years (or 104.32 months) to double.

$$2P = Pe^{0.08t}$$

$$2 = e^{0.08t}$$

$$\ln 2 = 0.08t$$

$$t = \frac{\ln 2}{0.08} \approx 8.66$$

Compounded continuously, it will take about 8.66 years (or 103.97 months) to double.

32. $2P = P\left(1+\frac{0.10}{12}\right)^{12t}$

$2 \approx (1.008333)^{12t}$

$\ln 2 \approx 12t\ \ln(1.008333)$

$t \approx \frac{\ln 2}{12\ \ln(1.008333)} \approx 6.96$

Compounded monthly, it will take about 6.96 years (or 83.52 months) to double.

$2P = Pe^{0.10t}$

$2 = e^{0.10t}$

$\ln 2 = 0.10t$

$t = \frac{\ln 2}{0.10} \approx 6.93$

Compounded continuously, it will take about 6.93 years (or 83.18 months) to double.

33. $150 = 100\left(1+\frac{0.08}{12}\right)^{12t}$

$1.5 \approx (1.006667)^{12t}$

$\ln 1.5 \approx 12t\ \ln(1.006667)$

$t \approx \frac{\ln 1.5}{12\ \ln(1.006667)} \approx 5.09$

Compounded monthly, it will take about 5.09 years (or 61.02 months).

$150 = 100e^{0.08t}$

$1.5 = e^{0.08t}$

$\ln 1.5 = 0.08t$

$t = \frac{\ln 1.5}{0.08} \approx 5.07$

Compounded continuously, it will take about 5.07 years (or 60.82 months).

34. $175 = 100\left(1+\frac{0.10}{12}\right)^{12t}$

$1.75 \approx (1.008333)^{12t}$

$\ln 1.75 \approx 12t\ \ln(1.008333)$

$t \approx \frac{\ln 1.75}{12\ \ln(1.008333)} \approx 5.62$

Compounded monthly, it will take about 5.62 years (or 67.43 months).

$175 = 100e^{0.10t}$

$1.75 = e^{0.10t}$

$\ln 1.75 = 0.10t$

$t = \frac{\ln 1.75}{0.10} \approx 5.60$

Compounded continuously, it will take about 5.60 years (or 67.15 months).

35. $25,000 = 10,000e^{0.06t}$

$2.5 = e^{0.06t}$

$\ln 2.5 = 0.06t$

$t = \frac{\ln 2.5}{0.06} \approx 15.27$

It will take about 15.27 years (or 15 years, 4 months).

36. $80,000 = 25,000e^{0.07t}$

$3.2 = e^{0.07t}$

$\ln 3.2 = 0.07t$

$t = \frac{\ln 3.2}{0.07} \approx 16.62$

It will take about 16.62 years (or 16 years, 8 months).

37. $A = 90,000(1+0.03)^5 = \$104,335$

The house will cost $104,335 in three years.

38. $A = 200(1+0.0125)^6 \approx \$215.48$

Her bill will be $215.48 after 6 months.

39. $P = 15,000e^{(-0.05)(3)} \approx \$12,910.62$

Jerome should ask for $12,910.62.

40. $P = 3,000\left(1+\frac{0.03}{12}\right)^{(-12)(0.5)} \approx \$2955.39$

John should save $2955.39.

41. $A = 15(1+0.15)^5 = 15(1.15)^5 \approx \$30.17$ per share for a total of about $3017.

42. $20 = 15(1+r)^2$

$$\frac{4}{3} = (1+r)^2$$

$$\sqrt{\frac{4}{3}} = 1+r$$

$$r = \sqrt{\frac{4}{3}} - 1 \approx 0.1547$$

$$r \approx 15.47\%$$

The annual return is approximately 15.47%.

43. $850,000 = 650,000(1+r)^3$

$$\frac{85}{65} = (1+r)^3$$

$$\sqrt[3]{\frac{85}{65}} = 1+r$$

$$r \approx \sqrt[3]{1.3077} - 1 \approx 0.0935$$

The annual return is approximately 9.35%.

44. $A = 5000(1+0.08)^{-10} = \$2,315.97$

The ring was valued at $2,315.97 ten years ago.

45. 5.6% compounded continuously:

$A = 1000e^{(0.056)(1)} = \$1057.60$

Jim will not have enough money to buy the computer.

5.9% compounded monthly:

$$A = 1000\left(1+\frac{0.059}{12}\right)^{12} = \$1060.62$$

The second bank offers the better deal.

46. 6.8% compounded continuously for 3 months:
Amount on April 1:

$A = 1000e^{(0.068)(0.25)} = \$1017.15$

5.25% compounded monthly for 1 month:
Amount on May 1

$$A = 1017.15\left(1+\frac{0.0525}{12}\right)^{(12)(1/12)} = \$1021.60$$

47. Will: 9% compounded semiannually:

$$A = 2000\left(1+\frac{0.09}{2}\right)^{(2)(20)} = \$11,632.73$$

Henry: 8.5% compounded continuously:

$A = 2000e^{(0.085)(20)} = \$10,947.89$

Will has more money after 20 years.

48. Value of $1000 compounded continuously at 10% for 3 years:

$A = 1000e^{(0.10)(3)} = \$1349.86$

April will have more money if she takes the $1000 now and invests it.

49. $P = 50,000;\ t = 5$

a. Simple interest at 12% per annum:

$A = 50,000 + 50,000(0.12)(5) = \$80,000$

$I = \$80,000 - \$50,000 = \$30,000$

b. 11.5% compounded monthly:

$$A = 50,000\left(1+\frac{0.115}{12}\right)^{(12)(5)} = \$88,613.59$$

$I = \$88,613.59 - \$50,000 = \$38,613.59$

c. 11.25% compounded continuously:

$A = 50,000e^{(0.1125)(5)} = \$87,752.73$

$I = \$87,752.73 - \$50,000 = \$37,752.73$

Thus, simple interest at 12% is the best option since it results in the least interest.

50. 360 day year:

$$r_e = \left(1+\frac{0.0425}{360}\right)^{360} - 1 \approx 0.043413439$$

The effective rate is about 4.34%.

365 day year:

$$r_e = \left(1+\frac{0.0425}{365}\right)^{365} - 1 \approx 0.043413475$$

The effective rate is about 4.34%.

51. a. $A = \$10,000,\ r = 0.10,\ n = 12,\ t = 20$

$$P = 10,000\left(1+\frac{0.10}{12}\right)^{(-12)(20)} \approx \$1364.62$$

b. $A = \$10,000,\ r = 0.10,\ t = 20$

$P = 10,000e^{(-0.10)(20)} \approx \$1353.35$

52. $A = \$40,000,\ r = 0.08,\ n = 1,\ t = 17$

$$P = 40,000\left(1+\frac{0.08}{1}\right)^{-17} \approx \$10,810.76$$

53. $A = \$10,000,\ r = 0.08,\ n = 1,\ t = 10$

$$P = 10,000\left(1+\frac{0.08}{1}\right)^{(-1)(10)} \approx \$4631.93$$

54. $A = \$25,000,\ \ P = 12,485.52,\ \ n = 1,\ t = 8$

$$25,000 = 12,485.52\left(1 + r^8\right)$$
$$\frac{25,000}{12,485.52} = (1 + r)^8$$
$$\sqrt[8]{\frac{25,000}{12,485.52}} = 1 + r$$
$$r = \sqrt[8]{\frac{25,000}{12,485.52}} - 1$$
$$r \approx 0.090665741$$

The annual rate of return is about 9.07%.

55. a. $t = \dfrac{\ln 2}{1 \cdot \ln\left(1 + \frac{0.12}{1}\right)} = \dfrac{\ln 2}{\ln(1.12)} \approx 6.12$ years

b. $t = \dfrac{\ln 3}{4 \cdot \ln\left(1 + \frac{0.06}{4}\right)} = \dfrac{\ln 3}{4\ln(1.015)} \approx 18.45$ years

c.
$$mP = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$m = \left(1 + \frac{r}{n}\right)^{nt}$$
$$\ln m = nt \cdot \ln\left(1 + \frac{r}{n}\right)$$
$$t = \frac{\ln m}{n \cdot \ln\left(1 + \frac{r}{n}\right)}$$

56. a. $t = \dfrac{\ln 8000 - \ln 1000}{0.10} \approx 20.79$ years

b.
$$35 = \frac{\ln 30,000 - \ln 2000}{r}$$
$$r = \frac{\ln 30,000 - \ln 2000}{35}$$
$$\approx 0.0774$$
$$r \approx 7.74\%$$

c.
$$A = Pe^{rt}$$
$$\frac{A}{P} = e^{rt}$$
$$\ln\left(\frac{A}{P}\right) = rt$$
$$\ln A - \ln P = rt$$
$$t = \frac{\ln A - \ln P}{r}$$

57. Answers will vary.

58. Answers will vary.

59. Answers will vary.

Section 4.8

1. $P(t) = 500e^{0.02t}$

a. $P(0) = 500e^{(0.02)\cdot(0)} = 500$ insects

b. growth rate = 2 %

c. $P(10) = 500e^{(0.02)\cdot(10)} \approx 611$ insects

d. Find t when $P = 800$:
$$800 = 500e^{0.02t}$$
$$1.6 = e^{0.02t}$$
$$\ln 1.6 = 0.02t$$
$$t = \frac{\ln 1.6}{0.02} \approx 23.5 \text{ days}$$

e. Find t when $P = 1000$:
$$1000 = 500e^{0.02t}$$
$$2 = e^{0.02t}$$
$$\ln 2 = 0.02t$$
$$t = \frac{\ln 2}{0.02} \approx 34.7 \text{ days}$$

2. $N(t) = 1000e^{0.01t}$

a. $N(0) = 1000e^{(0.01)\cdot(0)} = 1000$ bacteria

b. growth rate = 1 %

c. $N(4) = 1000e^{(0.01)\cdot(4)} \approx 1041$ bacteria

d. Find t when $N = 1700$:
$$1700 = 1000e^{0.01t}$$
$$1.7 = e^{0.01t}$$
$$\ln 1.7 = 0.01t$$
$$t = \frac{\ln 1.7}{0.01} \approx 53.1 \text{ hours}$$

e. Find t when $N = 2000$:

$$2000 = 1000e^{0.01t}$$
$$2 = e^{0.01t}$$
$$\ln 2 = 0.01t$$
$$t = \frac{\ln 2}{0.01} \approx 69.3 \text{ hours}$$

3. $A(t) = A_0 e^{-0.0244t} = 500e^{-0.0244t}$

a. decay rate = 2.44 %

b. $A(10) = 500e^{(-0.0244)(10)} \approx 391.7$ grams

c. Find t when $A = 400$:

$$400 = 500e^{-0.0244t}$$
$$0.8 = e^{-0.0244t}$$
$$\ln 0.8 = -0.0244t$$
$$t = \frac{\ln 0.8}{-0.0244} \approx 9.1 \text{ years}$$

d. Find t when $A = 250$:

$$250 = 500e^{-0.0244t}$$
$$0.5 = e^{-0.0244t}$$
$$\ln 0.5 = -0.0244t$$
$$t = \frac{\ln 0.5}{-0.0244} \approx 28.4 \text{ years}$$

4. $A(t) = A_0 e^{-0.087t} = 100e^{-0.087t}$

a. decay rate = 8.7 %

b. $A(9) = 100e^{(-0.087)(9)} \approx 45.7$ grams

c. Find t when $A = 70$:

$$70 = 100e^{-0.087t}$$
$$0.7 = e^{-0.087t}$$
$$\ln 0.7 = -0.087t$$
$$t = \frac{\ln 0.7}{-0.087} \approx 4.1 \text{ days}$$

d. Find t when $A = 50$:

$$50 = 100e^{-0.087t}$$
$$0.5 = e^{-0.087t}$$
$$\ln 0.5 = -0.087t$$
$$t = \frac{\ln 0.5}{-0.087} \approx 7.97 \text{ days}$$

5. Use $N(t) = N_0 e^{kt}$ and solve for k:

$$1800 = 1000e^{k(1)}$$
$$1.8 = e^k$$
$$k = \ln 1.8$$

When $t = 3$:

$$N(3) = 1000e^{(\ln 1.8)(3)} = 5832 \text{ mosquitos}$$

Find t when $N(t) = 10{,}000$:

$$10{,}000 = 1000e^{(\ln 1.8)t}$$
$$10 = e^{(\ln 1.8)t}$$
$$\ln 10 = (\ln 1.8)t$$
$$t = \frac{\ln 10}{\ln 1.8} \approx 3.9 \text{ days}$$

6. Use $N(t) = N_0 e^{kt}$ and solve for k:

$$800 = 500e^{k(1)}$$
$$1.6 = e^k$$
$$k = \ln 1.6$$

When $t = 5$: $N(5) = 500e^{(\ln 1.6)(5)} \approx 5243$ bacteria

Find t when $N(t) = 20{,}000$:

$$20{,}000 = 500e^{(\ln 1.6)t}$$
$$40 = e^{(\ln 1.6)t}$$
$$\ln 40 = (\ln 1.6)t$$
$$t = \frac{\ln 40}{\ln 1.6} \approx 7.85 \text{ hours}$$

7. Use $P(t) = P_0 e^{kt}$ and solve for k:

$$2P_0 = P_0 e^{k(1.5)}$$
$$2 = e^{1.5k}$$
$$\ln 2 = 1.5k$$
$$k = \frac{\ln 2}{1.5}$$

When $t = 2$:

$P(2) = 10{,}000e^{\left(\frac{\ln 2}{1.5}\right)(2)} \approx 25{,}198$ will be the population 2 years from now.

8. Use $P(t) = P_0 e^{kt}$ and solve for k:

$$800{,}000 = 900{,}000 e^{k(2)}$$
$$\frac{8}{9} = e^{2k}$$
$$\ln\left(\frac{8}{9}\right) = 2k$$
$$k = \frac{\ln(8/9)}{2}$$

When $t = 4$:

$P(4) = 900{,}000 e^{\left(\frac{\ln(8/9)}{2}\right)(4)} \approx 711{,}111$ will be the population in 1997.

9. Use $A = A_0 e^{kt}$ and solve for k:

$$0.5A_0 = A_0 e^{k(1690)}$$
$$0.5 = e^{1690k}$$
$$\ln 0.5 = 1690k$$
$$k = \frac{\ln 0.5}{1690}$$

When $A_0 = 10$ and $t = 50$:

$A = 10e^{\left(\frac{\ln 0.5}{1690}\right)(50)} \approx 9.797$ grams

10. Use $A = A_0 e^{kt}$ and solve for k:

$$0.5A_0 = A_0 e^{k\left(1.3\times10^9\right)}$$
$$0.5 = e^{\left(1.3\times10^9\right)k}$$
$$\ln 0.5 = 1.3\times10^9 k$$
$$k = \frac{\ln 0.5}{1.3\times10^9}$$

When $A_0 = 10$ and $t = 100$:

$A = 10e^{\left(\frac{\ln 0.5}{1.3\times10^9}\right)(100)} \approx 9.999999467$ grams

When $A_0 = 10$ and $t = 1000$:

$A = 10e^{\left(\frac{\ln 0.5}{1.3\times10^9}\right)(1000)} \approx 9.999994668$ grams

11. Use $A = A_0 e^{kt}$ and solve for k:
half-life $= 5600$ years

$$0.5A_0 = A_0 e^{k(5600)}$$
$$0.5 = e^{5600k}$$
$$\ln 0.5 = 5600k$$
$$k = \frac{\ln 0.5}{5600}$$

Solve for t when $A = 0.3A_0$:

$$0.3A_0 = A_0 e^{\left(\frac{\ln 0.5}{5600}\right)t}$$
$$0.3 = e^{\left(\frac{\ln 0.5}{5600}\right)t}$$
$$\ln 0.3 = \left(\frac{\ln 0.5}{5600}\right)t$$
$$t = \frac{\ln 0.3}{\left(\frac{\ln 0.5}{5600}\right)} \approx 9727$$

The tree died approximately 9727 years ago.

12. Use $A = A_0 e^{kt}$ and solve for k:
half-life $= 5600$ years

$$0.5A_0 = A_0 e^{k(5600)}$$
$$0.5A_0 = A_0 e^{k(5600)}$$
$$0.5 = e^{5600k}$$
$$\ln 0.5 = 5600k$$
$$k = \frac{\ln 0.5}{5600}$$

Solve for t when $A = 0.7A_0$:

$$0.7A_0 = A_0 e^{\left(\frac{\ln 0.5}{5600}\right)t}$$
$$0.7 = e^{\left(\frac{\ln 0.5}{5600}\right)t}$$
$$\ln 0.7 = \left(\frac{\ln 0.5}{5600}\right)t$$
$$t = \frac{\ln 0.7}{\frac{\ln 0.5}{5600}} \approx 2882$$

The fossil is about 2882 years old.

13. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 5$, $T = 70$, $u_0 = 450$, and $u = 300$:

$$300 = 70 + (450 - 70)e^{k(5)}$$
$$230 = 380e^{5k}$$
$$\frac{230}{380} = e^{5k}$$
$$\ln\left(\frac{23}{38}\right) = 5k$$
$$k = \frac{\ln\left(\frac{23}{38}\right)}{5} \approx -0.1004$$

$T = 70,\ u_0 = 450,\ u = 135$:

$$135 = 70 + (450 - 70)e^{\frac{\ln(23/38)}{5}t}$$
$$65 = 380e^{\frac{\ln(23/38)}{5}t}$$
$$\frac{65}{380} = e^{\frac{\ln(23/38)}{5}t}$$
$$\ln\left(\frac{65}{380}\right) = \frac{\ln(23/38)}{5}t$$
$$t = \frac{\ln(65/380)}{\left(\frac{\ln(23/38)}{5}\right)} \approx 18 \text{ minutes}$$

The pizza will be cool enough to eat at about 5:18 PM.

b. $T = 70,\ u_0 = 450,\ u = 160$:

$$160 = 70 + (450 - 70)e^{\frac{\ln(23/38)}{5}t}$$
$$90 = 380e^{\frac{\ln(23/38)}{5}t}$$
$$\frac{90}{380} = e^{\frac{\ln(23/38)}{5}t}$$
$$\ln\left(\frac{90}{380}\right) = \frac{\ln(23/38)}{5}t$$
$$t = \frac{\ln(90/380)}{\left(\frac{\ln(23/38)}{5}\right)} \approx 14.3 \text{ minutes}$$

The pizza will be 160˚F after about 14.3 minutes.

c. As time passes, the temperature gets closer to 70˚F.

14. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 2$, $T = 38$, $u_0 = 72$, and $u = 60$:

$$60 = 38 + (72 - 38)e^{k(2)}$$
$$22 = 34e^{2k}$$
$$\frac{22}{34} = e^{2k}$$
$$\ln\left(\frac{22}{34}\right) = 2k$$
$$k = \frac{\ln(22/34)}{2}$$

$T = 38,\ u_0 = 72,\ t = 7$

$$u = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)(7)}$$
$$u = 38 + 34e^{\left(\frac{\ln(22/34)}{2}\right)(7)} \approx 45.41^\circ \text{F}$$

After 7 minutes the thermometer will read about 45.41˚F.

b. Find t when $u = 39^\circ$ F

$$39 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$1 = 34e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$\frac{1}{34} = e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$\ln\left(\frac{1}{34}\right) = \left(\frac{\ln(22/34)}{2}\right)t$$
$$t = \frac{\ln(1/34)}{\left(\frac{\ln(22/34)}{2}\right)} \approx 16.2$$

The thermometer will read 39 degrees after about 16.2 minutes.

c. $T = 38,\ u_0 = 72,\ u = 45$:

$$45 = 38 + (72 - 38)e^{\frac{\ln(22/34)}{2}t}$$
$$7 = 34e^{\frac{\ln(22/34)}{2}t}$$
$$\frac{7}{34} = e^{\frac{\ln(22/34)}{2}t}$$
$$\ln\left(\frac{7}{34}\right) = \frac{\ln(22/34)}{2}t$$
$$t = \frac{\ln(7/34)}{\left(\frac{\ln(22/34)}{2}\right)} \approx 7.26 \text{ minutes}$$

The thermometer will read $45°F$ after about 7.26 minutes.

d. As time passes, the temperature gets closer to 38˚F.

15. Using $u = T + (u_0 - T)e^{kt}$ with $t = 3$, $T = 35$, $u_0 = 8$, and $u = 15$:

$$15 = 35 + (8 - 35)e^{k(3)}$$
$$-20 = -27e^{3k}$$
$$\frac{20}{27} = e^{3k}$$
$$\ln\left(\frac{20}{27}\right) = 3k$$
$$k = \frac{\ln(20/27)}{3}$$

At $t = 5$:

$$u = 35 + (8 - 35)e^{\left(\frac{\ln(20/27)}{3}\right)(5)} \approx 18.63°\text{C}$$

After 5 minutes, the thermometer will read approximately $18.63°\text{C}$.

At $t = 10$:

$$u = 35 + (8 - 35)e^{\left(\frac{\ln(20/27)}{3}\right)(10)} \approx 25.1°\text{C}$$

After 10 minutes, the thermometer will read approximately $25.1°\text{C}$

16. Using $u = T + (u_0 - T)e^{kt}$ with $t = 10$, $T = 70$, $u_0 = 28$, and $u = 35$:

$$35 = 70 + (28 - 70)e^{k(10)}$$
$$-35 = -42e^{10k}$$
$$\frac{35}{42} = e^{10k}$$
$$\ln\left(\frac{35}{42}\right) = 10k$$
$$k = \frac{\ln(35/42)}{10}$$

At $t = 30$:

$$u = 70 + (28 - 70)e^{\left(\frac{\ln(35/42)}{10}\right)(30)} \approx 45.69°\text{F}$$

After 30 minutes, the temperature of the steaks will be approximately $45.69°F$.

Find the value of t so that the u = 45˚F:

$$45 = 70 + (28 - 70)e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$-25 = -42e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$\frac{25}{42} = e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$\ln\left(\frac{25}{42}\right) = \left(\frac{\ln(35/42)}{10}\right)t$$
$$t = \frac{\ln(25/42)}{\left(\frac{\ln(35/42)}{10}\right)} \approx 28.46$$

The temperature of the steaks will be $45°F$ after about 28.46 minutes.

17. Use $A = A_0e^{kt}$ and solve for k:

$$15 = 25e^{k(10)}$$
$$0.6 = e^{10k}$$
$$\ln 0.6 = 10k$$
$$k = \frac{\ln 0.6}{10}$$

When $A_0 = 25$ and $t = 24$:

$$A = 25e^{\left(\frac{\ln 0.6}{10}\right)(24)} \approx 7.34$$

There will be about 7.34 kilograms of salt left after 1 day.

Find t when $A = 0.5A_0$:

$$0.5 = 25e^{\left(\frac{\ln 0.6}{10}\right)t}$$

$$0.02 = e^{\left(\frac{\ln 0.6}{10}\right)t}$$

$$\ln 0.02 = \left(\frac{\ln 0.6}{10}\right)t$$

$$t = \frac{\ln 0.02}{\left(\frac{\ln 0.6}{10}\right)} \approx 76.6$$

It will take about 76.6 hours until ½ kilogram of salt is left.

18. Use $A = A_0e^{kt}$ with $A_0 = 40$, $A = 10$, and $t = 2$, solve for k:

$$10 = 40e^{k(2)}$$

$$0.25 = e^{2k}$$

$$\ln 0.25 = 2k$$

$$k = \frac{\ln 0.25}{2}$$

At $t = 5$: then $A = 40e^{\left(\frac{\ln 0.25}{2}\right)(5)} \approx 1.25$

After 5 seconds, the voltage will be about 1.25 volts.

19. Use $A = A_0e^{kt}$ and solve for k:

$$0.5A_0 = A_0e^{k(8)}$$

$$0.5 = e^{8k}$$

$$\ln 0.5 = 8k$$

$$k = \frac{\ln 0.5}{8}$$

Find t when $A = 0.1A_0$:

$$0.1A_0 = A_0e^{\left(\frac{\ln 0.5}{8}\right)t}$$

$$0.1 = e^{\left(\frac{\ln 0.5}{8}\right)t}$$

$$\ln 0.1 = \left(\frac{\ln 0.5}{8}\right)t$$

$$t = \frac{\ln 0.1}{\left(\frac{\ln 0.5}{8}\right)} \approx 26.6$$

The farmers need to wait about 26.6 days before using the hay.

20. Using $u = T + (u_0 - T)e^{kt}$ with $t = 2$, $T = 325$, $u_0 = 75$, and $u = 100$:

$$100 = 325 + (75 - 325)e^{k(2)}$$

$$-225 = -250e^{2k}$$

$$0.9 = e^{2k}$$

$$2k = \ln 0.9$$

$$k = \frac{\ln 0.9}{2}$$

Find the value of t so that u = 175˚F:

$$175 = 325 + (75 - 325)e^{\left(\frac{\ln 0.9}{2}\right)t}$$

$$-150 = -250e^{\left(\frac{\ln 0.9}{2}\right)t}$$

$$0.6 = e^{\left(\frac{\ln 0.9}{2}\right)t}$$

$$\ln 0.6 = \left(\frac{\ln 0.9}{2}\right)t$$

$$t = \frac{\ln 0.6}{\left(\frac{\ln 0.9}{2}\right)} \approx 9.7$$

The hotel may serve their guests about 9.7 hours after noon or at about 9:42 PM.

21. **a.** The maximum proportion is the carrying capacity, c = 0.9 = 90%.

b. $P(0) = \dfrac{0.9}{1 + 6e^{-0.32(0)}} = \dfrac{0.9}{1 + 6 \cdot 1} = \dfrac{0.9}{7} = 0.1286$

In 2000, about 12.86% of U.S. households owned a DVD player.

c. $t = 2005 - 2000 = 5$

$$P(5) = \frac{0.9}{1 + 6e^{-0.32(5)}} = \frac{0.9}{1 + 6e^{-1.6}} \approx 0.4070$$

In 2005, about40.7% of U.S. households owned a DVD player.

d. We need to find t such that $P = 0.8$.

$$0.8 = \frac{0.9}{1+6e^{-0.32t}}$$
$$1+6e^{-0.32t} = \frac{9}{8}$$
$$6e^{-0.32t} = \frac{1}{8}$$
$$e^{-0.32t} = \frac{1}{48}$$
$$-0.32t = \ln(1/48)$$
$$t = \frac{\ln(1/48)}{-0.32} \approx 12.1$$

Since $2000+12.1 = 2012.1$, 80% of households will own a DVD player in 2012.

e. We need to find t such that $P = 0.45$.

$$0.45 = \frac{0.9}{1+6e^{-0.32t}}$$
$$1+6e^{-0.32t} = 2$$
$$6e^{-0.32t} = 1$$
$$e^{-0.32t} = \frac{1}{6}$$
$$-0.32t = \ln(1/6)$$
$$t = \frac{\ln(1/6)}{-0.32} \approx 5.6$$

About 5.6 years after 2000 (i.e. in about mid 2005), 45% of the population owned a DVD player.

22. a. The maximum proportion is the carrying capacity, $c = 0.90 = 90\%$.

b. $$P(0) = \frac{0.90}{1+3.5e^{-0.339(0)}} = \frac{0.90}{1+3.5\cdot 1} = \frac{0.90}{4.5} = 0.2$$

When it is first introduced, Intel's latest coprocessor will be in 20% of computers sold at Best Buy.

c. $$P(4) = \frac{0.90}{1+3.5e^{-0.339(4)}} = \frac{0.90}{1+3.5e^{-1.356}} \approx 0.473$$

After 4 months, Intel's latest coprocessor will be in about 47.3% of PCs sold at Best Buy.

d. We need to find t such that $P = 0.75$.

$$0.75 = \frac{0.90}{1+3.5e^{-0.339t}}$$
$$1+3.5e^{-0.339t} = 1.2$$
$$3.5e^{-0.339t} = 0.2$$
$$e^{-0.339t} = \frac{2}{35}$$
$$-0.339t = \ln(2/35)$$
$$t = \frac{\ln(2/35)}{-0.339} \approx 8.44$$

75% of PCs sold at Best Buy have Intel's latest coprocessor about 8.44 months after it has been introduced.

e. We need to find t such that $P = 0.45$.

$$0.45 = \frac{0.90}{1+3.5e^{-0.339t}}$$
$$1+3.5e^{-0.339t} = 2$$
$$3.5e^{-0.339t} = 1$$
$$e^{-0.339t} = \frac{2}{7}$$
$$-0.339t = \ln(2/7)$$
$$t = \frac{\ln(2/7)}{-0.339} \approx 3.70$$

45% of PCs sold at Best Buy have Intel's latest coprocessor about 3.7 months after it has been introduced.

23. a. As $t \to \infty$, $e^{-0.439t} \to 0$. Thus, $P(t) \to 1000$. The carrying capacity is 1000 grams of bacteria.

b. Growth rate = 0.439 = 43.9%.

c. $$P(0) = \frac{1000}{1+32.33e^{-0.439(0)}} = \frac{1000}{33.33} = 30$$

The initial population was 30 grams of bacteria.

d. $$P(9) = \frac{1000}{1+32.33e^{-0.439(9)}} \approx 616.6$$

After 9 hours, the population of bacteria will be about 616.6 grams.

e. We need to find t such that $P = 700$:

$$\begin{aligned} 700 &= \frac{1000}{1+32.33e^{-0.439t}} \\ 1+32.33e^{-0.439t} &= \frac{10}{7} \\ 32.33e^{-0.439t} &= \frac{3}{7} \\ e^{-0.439t} &= \frac{3}{226.31} \\ -0.439t &= \ln(3/226.31) \\ t &= \frac{\ln(3/226.31)}{-0.439} \approx 9.85 \end{aligned}$$

The population of bacteria will be 700 grams after about 9.85 hours.

f. We need to find t such that $P = \frac{1}{2}(1000) = 500$:

$$\begin{aligned} 500 &= \frac{1000}{1+32.33e^{-0.439t}} \\ 1+32.33e^{-0.439t} &= 2 \\ 32.33e^{-0.439t} &= 1 \\ e^{-0.439t} &= \frac{1}{32.33} \\ -0.439t &= \ln(1/32.33) \\ t &= \frac{\ln(1/32.33)}{-0.439} \approx 7.9 \end{aligned}$$

The population of bacteria will reach one-half of is carrying capacity after about 7.9 hours.

24. a. As $t \to \infty$, $e^{-0.162t} \to 0$. Thus, $P(t) \to 500$. The carrying capacity is 500 bald eagles.

b. Growth rate = 0.162 = 16.2%.

c. $P(3) = \dfrac{500}{1+83.33e^{-0.162(3)}} \approx 9.56$

After 3 years, the population is almost 10 bald eagles.

d. We need to find t such that $P = 300$:

$$\begin{aligned} 300 &= \frac{500}{1+83.33e^{-0.162t}} \\ 1+83.33e^{-0.162t} &= \frac{5}{3} \\ 83.33e^{-0.162t} &= \frac{2}{3} \\ e^{-0.162t} &\approx 0.008 \\ -0.162t &= \ln(0.008) \\ t &= \frac{\ln(0.008)}{-0.162} \approx 29.8 \end{aligned}$$

The bald eagle population will be 300 in approximately 29.8 years.

e. We need to find t such that $P = \frac{1}{2}(500) = 250$:

$$\begin{aligned} 250 &= \frac{500}{1+83.33e^{-0.162t}} \\ 1+83.33e^{-0.162t} &= 2 \\ 83.33e^{-0.162t} &= 1 \\ e^{-0.162t} &\approx 0.012 \\ -0.162t &= \ln(0.012) \\ t &= \frac{\ln(0.012)}{-0.162} \approx 27.3 \end{aligned}$$

The bald eagle population will reach one-half of its carrying capacity after about 27.3 years.

25. a. $y = \dfrac{6}{1+e^{-(5.085-0.1156(100))}} \approx 0.0092$

At $100°F$, the predicted number of eroded or leaky primary O-rings will be about 0.

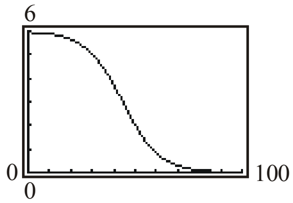
b. $y = \dfrac{6}{1+e^{-(5.085-0.1156(60))}} \approx 0.81$

At $60°F$, the predicted number of eroded or leaky primary O-rings will be about 1.

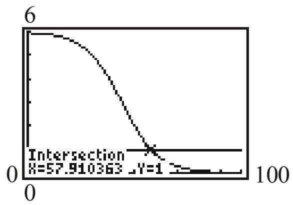
c. $y = \dfrac{6}{1+e^{-(5.085-0.1156(30))}} \approx 5.01$

At $30°F$, the predicted number of eroded or leaky primary O-rings will be about 5.

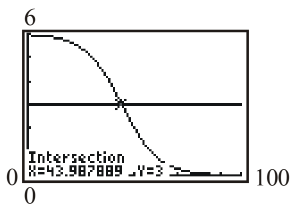
d. $Y_1 = \dfrac{6}{1+e^{-(5.085-0.1156x)}}$



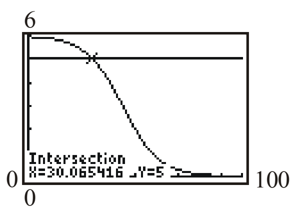
Use INTERSECT with $Y_2 = 1,\ 3,\ \text{and } 5$:



The predicted number of eroded or leaky O-rings is 1 when the temperature is about $57.91°F$.



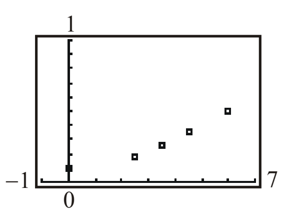
The predicted number of eroded or leaky O-rings is 3 when the temperature is about $43.99°F$.



The predicted number of eroded or leaky O-rings is 5 when the temperature is about $37.07°F$.

Section 4.9

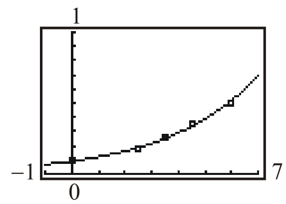
1. a.



b. Using EXPonential REGression on the data yields: $y = 0.0903(1.3384)^x$

c.
$$\begin{aligned} y &= 0.0903(1.3384)^x \\ &= 0.0903\left(e^{\ln(1.3384)}\right)^x \\ &= 0.0903e^{\ln(1.3384)x} \\ N(t) &= 0.0903e^{0.2915t} \end{aligned}$$

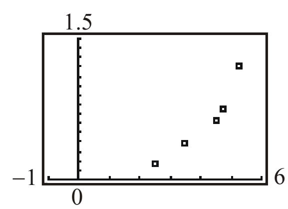
d. $Y_1 = 0.0903e^{0.2915x}$



e. $N(7) = 0.0903e^{(0.2915)\cdot 7} \approx 0.69$ bacteria

f. We need to find t when $N = 0.75$:
$$\begin{aligned} 0.0903e^{(0.2915)\cdot t} &= 0.75 \\ e^{(0.2915)\cdot t} &= \frac{0.75}{0.0903} \\ 0.2915t &= \ln\left(\frac{0.75}{0.0903}\right) \\ t &\approx \frac{\ln\left(\frac{0.75}{0.0903}\right)}{0.2915} \approx 7.26 \text{ hours} \end{aligned}$$

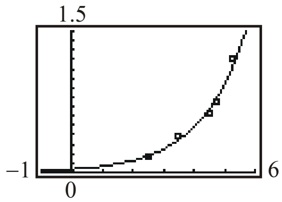
2. a.



b. Using EXPonential REGression on the data yields: $y = 0.0339(1.9474)^x$

c.
$$\begin{aligned} y &= 0.0339(1.9474)^x \\ &= 0.0339\left(e^{\ln(1.9474)}\right)^x \\ &= 0.0339e^{\ln(1.9474)x} \\ N(t) &= 0.0339e^{(0.6665)t} \end{aligned}$$

d. $Y_1 = 0.0339e^{(0.6665)x}$

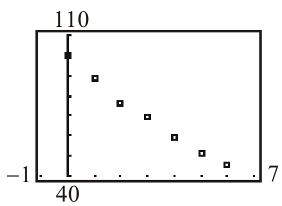


e. $N(6) = 0.0339e^{(0.6665)\cdot 6} \approx 1.85$ bacteria

f. We need to find t when $N = 2.1$:

$$0.0339e^{(0.6665)t} = 2.1$$
$$e^{(0.6665)t} = \frac{2.1}{0.0339}$$
$$0.6665t = \ln\left(\frac{2.1}{0.0339}\right)$$
$$t \approx \frac{\ln\left(\frac{2.1}{0.0339}\right)}{0.6665} \approx 6.19 \text{ hours}$$

3. a.

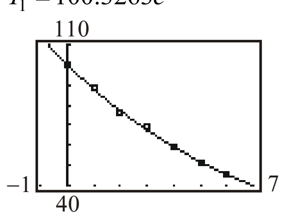


b. Using EXPonential REGression on the data yields: $y = 100.3263(0.8769)^x$

c. $y = 100.3263(0.8769)^x$

$$= 100.3263\left(e^{\ln(0.8769)}\right)^x$$
$$= 100.3263e^{\ln(0.8769)x}$$
$$A(t) = 100.3263e^{(-0.1314)t}$$

d. $Y_1 = 100.3263e^{(-0.1314)x}$



e. We need to find t when $A(t) = 0.5 \cdot A_0$

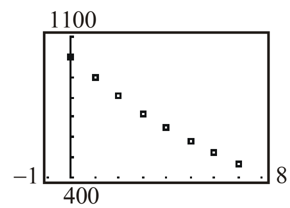
$$100.3263e^{(-0.1314)t} = (0.5)(100.3263)$$
$$e^{(-0.1314)t} = 0.5$$
$$-0.1314t = \ln 0.5$$
$$t = \frac{\ln 0.5}{-0.1314} \approx 5.3 \text{ weeks}$$

f. $A(50) = 100.3263e^{(-0.1314)\cdot 50} \approx 0.14$ grams

g. We need to find t when $A(t) = 20$.

$$100.3263e^{(-0.1314)t} = 20$$
$$e^{(-0.1314)t} = \frac{20}{100.3263}$$
$$-0.1314t = \ln\left(\frac{20}{100.3263}\right)$$
$$t = \frac{\ln\left(\frac{20}{100.3263}\right)}{-0.1314}$$
$$\approx 12.3 \text{ weeks}$$

4. a.

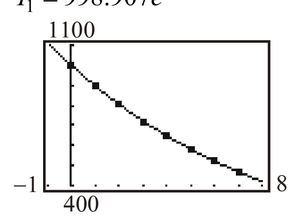


b. Using EXPonential REGression on the data yields: $y = 998.907(0.8976)^x$

c. $y = 998.907(0.8976)^x$

$$= 998.907\left(e^{\ln(0.8976)}\right)^x$$
$$= 998.907e^{\ln(0.8976)x}$$
$$A(t) = 998.907e^{(-0.1080)t}$$

d. $Y_1 = 998.907e^{(-0.1080)x}$



e. We need to find t when $A(t) = 0.5 \cdot A_0$

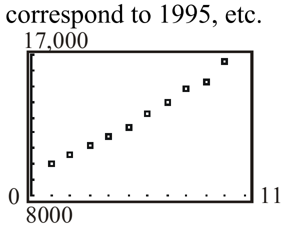
$$998.907e^{(-0.1080)t} = (0.5)(998.907)$$
$$e^{(-0.1080)t} = 0.5$$
$$-0.1080t = \ln 0.5$$
$$t = \frac{\ln 0.5}{-0.1080} \approx 6.42 \text{ days}$$

f. $A(20) = 998.907e^{(-0.1080)\cdot 20} \approx 115$ grams

g. We need to find t when $A = 200$:

$$998.907e^{(-0.1080)t} = 200$$
$$e^{(-0.1080)t} = \frac{200}{998.907}$$
$$-0.1080t = \ln\left(\frac{200}{998.907}\right)$$
$$t = \frac{\ln\left(\frac{200}{998.907}\right)}{-0.1080} \approx 14.9 \text{ days}$$

5. a. Let $x = 1$ correspond to 1994, $x = 2$ correspond to 1995, etc.



b. Using EXPonential REGression on the data yields: $y = 9478.4453(1.056554737)^x$

c. The average annual rate of return over the 10 years is $1.056554737 - 1 \approx .0566 = 5.66\%$.

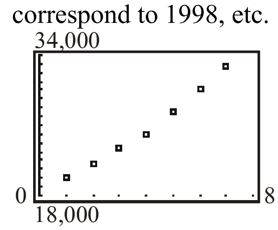
d. The year 2021 corresponds to $x = 28$, so $y = 9478.4453(1.056554737)^{28} \approx 44,229.61$. The value of the account in 2021 is about \$44,229.61.

e. We need to find x when $y = 50,000$:

$$9478.4453(1.056554737)^x = 50,000$$
$$(1.056554737)^x = \frac{50,000}{9478.4453}$$
$$x \ln 1.056554737 = \ln\left(\frac{50,000}{9478.4453}\right)$$
$$x = \frac{\ln\left(\frac{50,000}{9478.4453}\right)}{\ln 1.056554737} \approx 30.23$$

The account will be worth \$50,000 early in the year 2023.

6. a. Let $x = 1$ correspond to 1997, $x = 2$ correspond to 1998, etc.



b. Using EXPonential REGression on the data yields: $y = 18,258.1235(1.0855)^x$

c. The average annual rate of return over 7 years is $1.0855 - 1 = .0855 = 8.55\%$

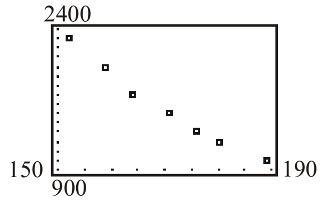
d. The year 2020 corresponds to $x = 24$, so $y = 18,258.1235(1.0855)^{24} \approx \$130,789.24$.

e. We need to find x when $y = 80,000$:

$$(18,258.1235)(1.0855)^x = 80,000$$
$$(1.0855)^x = \frac{80,000}{18,258.1235}$$
$$x \ln 1.0855 = \ln\left(\frac{80,000}{18,258.1235}\right)$$
$$x = \frac{\ln\left(\frac{80,000}{18,258.1235}\right)}{\ln 1.0855} \approx 18.01$$

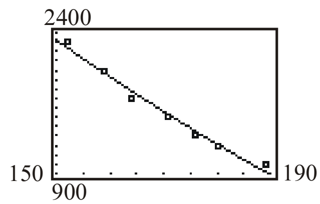
The account will be worth \$80,000 in the beginning of 2014.

7. a.



b. Using LnREGression on the data yields:
$y = 32,741.02 - 6070.96 \ln x$

c. $Y_1 = 32,741.02 - 6070.96 \ln x$

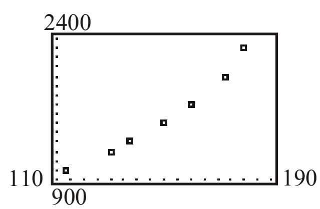


d. We need to find x when $y = 1650$:

$$\begin{aligned} 1650 &= 32,741.02 - 6070.96 \ln x \\ -31,091.02 &= -6070.96 \ln x \\ \frac{-31,091.02}{-6070.96} &= \ln x \\ 5.1213 &\approx \ln x \\ e^{5.1213} &\approx x \\ x &\approx 168 \end{aligned}$$

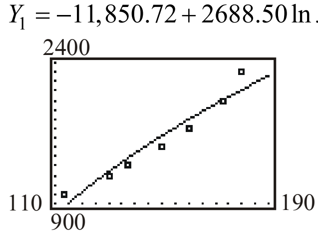
If the price were \$1650, then approximately 168 computers would be demanded.

8. a.



b. Using LnREGression on the data yields: $y = -11,850.72 + 2688.50 \ln x$

c. $Y_1 = -11,850.72 + 2688.50 \ln x$

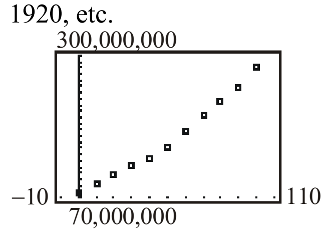


d. Find x when $y = 1650$:

$$\begin{aligned} 1650 &= -11,850.72 + 2688.50 \ln x \\ 13,500.72 &= 2688.50 \ln x \\ \frac{13,500.72}{2688.50} &= \ln x \\ 5.0216 &\approx \ln x \\ e^{5.0216} &\approx x \\ x &\approx 152 \end{aligned}$$

If the price were \$1650, then approximately 152 computers would be demanded.

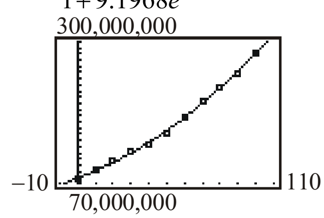
9. a. Let $x = 0$ correspond to 1900, $x = 10$ correspond to 1910, $x = 20$ correspond to 1920, etc.



b. Using LOGISTIC REGression on the data yields:

$$y = \frac{799,475,916.5}{1 + 9.1968e^{-0.01603x}}$$

c. $Y_1 = \dfrac{799,475,916.5}{1 + 9.1968e^{-0.01603x}}$:



d. As $x \to \infty$, $9.1968e^{-0.01603x} \to 0$, which means $1 + 9.1968e^{-0.01603x} \to 1$, so

$$y = \frac{799,475,916.5}{1 + 9.1968e^{-0.01603x}} \to 799,475,916.5$$

Therefore, the carrying capacity of the United States is approximately 799,475,917 people.

e. The year 2004 corresponds to $x = 104$, so

$$y = \frac{799,475,916.5}{1 + 9.1968e^{-0.01603(104)}} \approx 292,183,980 \text{ people}$$

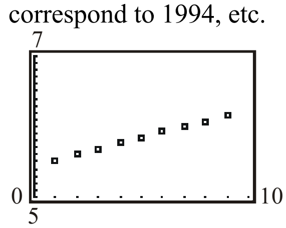
f. Find
x when $y = 300,000,000$

$$\frac{799,475,916.5}{1+9.1968e^{-0.01603x}} = 300,000,000$$
$$799,475,916.5 = 300,000,000\left(1+9.1968e^{-0.01603x}\right)$$
$$\frac{799,475,916.5}{300,000,000} = 1+9.1968e^{-0.01603x}$$

$$\frac{799,475,916.5}{300,000,000} - 1 = 9.1968e^{-0.01603x}$$
$$1.6649 \approx 9.1968e^{-0.01603x}$$
$$\frac{1.6649}{9.1968} \approx e^{-0.01603x}$$
$$\ln\left(\frac{1.6649}{9.1968}\right) \approx -0.01603x$$
$$\frac{\ln\left(\frac{1.6649}{9.1968}\right)}{-0.01603} \approx x$$
$$x \approx 107$$

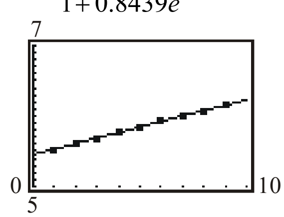
Therefore, the United States population will be 300,000,000 around the year 2007.

10. a. Let $x = 1$ correspond to 1993, $x = 2$ correspond to 1994, etc.



(b) Using LOGISTIC REGression on the data yields: $y = \dfrac{10.05267}{1+0.8439e^{-0.0320x}}$

c. $Y_1 = \dfrac{10.05267}{1+0.8439e^{-0.0320x}}$:

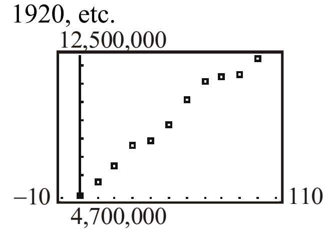


d. As $x \to \infty$, $0.8439e^{-0.0320x} \to 0$, which means $1+0.8439e^{-0.0320x} \to 1$, so
$$y = \frac{10.05267}{1+0.8439e^{-0.0320x}} \to 10.05267$$
Therefore, the carrying capacity of the world is approximately 10.053 billion people.

e. The year 2004 corresponds to $x = 12$, so
$$y = \frac{10.05267}{1+0.8439e^{-0.0320(12)}} \approx 6.38.$$
In 2004, the population of the world was approximately 6.38 billion people.

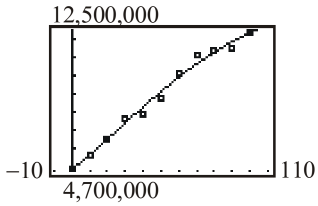
f. We need to find x when $y = 7$:
$$\frac{10.05267}{1+0.8439e^{-0.0320x}} = 7$$
$$10.05267 = 7\left(1+0.8439e^{-0.0320x}\right)$$
$$\frac{10.05267}{7} = 1+0.8439e^{-0.0320x}$$
$$\frac{10.05267}{7} - 1 = 0.8439e^{-0.0320x}$$
$$0.4361 \approx 0.8439e^{-0.0320x}$$
$$\frac{0.4361}{0.8439} \approx e^{-0.0320x}$$
$$\ln\left(\frac{0.4361}{0.8439}\right) \approx -0.0320x$$
$$\frac{\ln\left(\frac{0.4361}{0.8439}\right)}{-0.0320} \approx x$$
$$x \approx 21$$
Therefore, the world population will be 7 billion in approximately the year 2013.

11. a. Let $x = 0$ correspond to 1900, $x = 10$ correspond to 1910, $x = 20$ correspond to 1920, etc.



b. Using LOGISTIC REGression on the data yields: $y = \dfrac{14,471,245.24}{1+2.01527e^{-0.02458x}}$

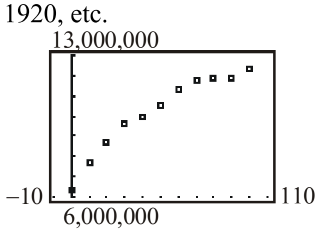
c. $Y_1 = \dfrac{14,471,245.24}{1+2.01527e^{-0.02458x}}$



d. As $x \to \infty$, $2.01527e^{-0.02458x} \to 0$, which means $1+2.01527e^{-0.02458x} \to 1$, so
$y = \dfrac{14,471,245.24}{1+2.01527e^{-0.02458x}} \to 14,471,245.24$
Therefore, the carrying capacity of Illinois is approximately 14,471,245 people.

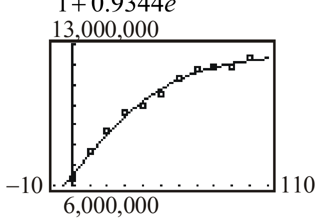
e. The year 2010 corresponds to $x = 110$, so
$y = \dfrac{14,471,245.24}{1+2.01527e^{-0.02458(110)}} \approx 12,750,854$.
In 2010, the population of Illinois will be approximately 12,750,854 people.

12. a. Let $x = 0$ correspond to 1900, $x = 10$ correspond to 1910, $x = 20$ correspond to 1920, etc.



b. Using LOGISTIC REGression on the data yields: $y = \dfrac{12,529,186.49}{1+0.9344e^{-0.03492x}}$

c. $Y_1 = \dfrac{12,529,186.49}{1+0.9344e^{-0.3492x}}$:



d. As $x \to \infty$, $0.9344e^{-0.3492x} \to 0$, which means $1+0.9344e^{-0.3492x} \to 1$, so
$y = \dfrac{12,529,186.49}{1+0.9344e^{-0.3492x}} \to 12,529,186.49$.
Therefore, the carrying capacity of Pennsylvania is approximately 12,529,186 people.

e. The year 2010 corresponds to $x = 110$, so
$y = \dfrac{12,529,186.49}{1+0.9344e^{-0.03492(110)}} \approx 12,282,799$
In 2010, the population of Pennsylvania will be approximately 12,282,799 people.

Chapter 4 Review Exercises

1. $f(x) = 3x-5 \qquad g(x) = 1-2x^2$

a.
$$\begin{aligned}(f \circ g)(2) &= f(g(2))\\ &= f\left(1-2(2)^2\right)\\ &= f(-7)\\ &= 3(-7)-5\\ &= -26\end{aligned}$$

b.
$$\begin{aligned}(g \circ f)(-2) &= g(f(-2))\\ &= g\left(3(-2)-5\right)\\ &= g(-11)\\ &= 1-2(-11)^2\\ &= -241\end{aligned}$$

c.
$$\begin{aligned}(f \circ f)(4) &= f(f(4))\\ &= f\left(3(4)-5\right)\\ &= f(7)\\ &= 3(7)-5\\ &= 16\end{aligned}$$

d.
$$\begin{aligned}(g \circ g)(-1) &= g(g(-1))\\ &= g\left(1-2(-1)^2\right)\\ &= g(-1)\\ &= 1-2(-1)^2\\ &= -1\end{aligned}$$

2. $f(x) = 4 - x \qquad g(x) = 1 + x^2$

a. $$\begin{aligned}(f \circ g)(2) &= f(g(2)) \\ &= f\left(1 + 2^2\right) \\ &= f(5) \\ &= 4 - 5 \\ &= -1\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(4 - (-2)\right) \\ &= g(6) \\ &= 1 + 6^2 \\ &= 37\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(4) &= f(f(4)) \\ &= f\left(4 - 4\right) \\ &= f(0) \\ &= 4 - 0 \\ &= 4\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(-1) &= g(g(-1)) \\ &= g\left(1 + (-1)^2\right) \\ &= g(2) \\ &= 1 + 2^2 \\ &= 5\end{aligned}$$

3. $f(x) = \sqrt{x+2} \qquad g(x) = 2x^2 + 1$

a. $$\begin{aligned}(f \circ g)(2) &= f(g(2)) \\ &= f\left(2(2)^2 + 1\right) \\ &= f(9) \\ &= \sqrt{9+2} \\ &= \sqrt{11}\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(\sqrt{-2+2}\right) \\ &= g(0) \\ &= 2(0)^2 + 1 \\ &= 1\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(4) &= f(f(4)) \\ &= f\left(\sqrt{4+2}\right) \\ &= f\left(\sqrt{6}\right) \\ &= \sqrt{\sqrt{6} + 2}\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(-1) &= g(g(-1)) \\ &= g\left(2(-1)^2 + 1\right) \\ &= g(3) \\ &= 2(3)^2 + 1 \\ &= 19\end{aligned}$$

4. $f(x) = 1 - 3x^2 \qquad g(x) = \sqrt{4 - x}$

a. $$\begin{aligned}(f \circ g)(2) &= f(g(2)) \\ &= f\left(\sqrt{4-2}\right) \\ &= f\left(\sqrt{2}\right) \\ &= 1 - 3\left(\sqrt{2}\right)^2 \\ &= 1 - 3 \cdot 2 \\ &= -5\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(1 - 3(-2)^2\right) \\ &= g(-11) \\ &= \sqrt{4 - (-11)} \\ &= \sqrt{15}\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(4) &= f(f(4)) \\ &= f\left(1 - 3(4)^2\right) \\ &= f\left(-47\right) \\ &= 1 - 3(-47)^2 \\ &= -6626\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(-1) &= g(g(-1)) \\ &= g\left(\sqrt{4 - (-1)}\right) \\ &= g\left(\sqrt{5}\right) \\ &= \sqrt{4 - \sqrt{5}}\end{aligned}$$

5. $f(x) = e^x \qquad g(x) = 3x - 2$

a. $$\begin{aligned}(f \circ g)(2) &= f(g(2)) \\ &= f\left(3(2) - 2\right) \\ &= f(4) \\ &= e^4\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(e^{-2}\right) \\ &= 3e^{-2} - 2 \\ &= \frac{3}{e^2} - 2\end{aligned}$$

c. $(f \circ f)(4) = f(f(4)) = f\left(e^4\right) = e^{e^4}$

d. $$\begin{aligned}(g \circ g)(-1) &= g(g(-1)) \\ &= g\left(3(-1)-2\right) \\ &= g(-5) \\ &= 3(-5)-2 \\ &= -17\end{aligned}$$

6. $f(x) = \dfrac{2}{1+2x^2} \qquad g(x) = 3x$

a. $$\begin{aligned}(f \circ g)(2) &= f(g(2)) \\ &= f\left(3(2)\right) \\ &= f(6) \\ &= \frac{2}{1+2(6)^2} \\ &= \frac{2}{73}\end{aligned}$$

b. $$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(\frac{2}{1+2(-2)^2}\right) \\ &= g\left(\frac{2}{9}\right) \\ &= 3\left(\frac{2}{9}\right) \\ &= \frac{2}{3}\end{aligned}$$

c. $$\begin{aligned}(f \circ f)(4) &= f(f(4)) \\ &= f\left(\frac{2}{1+2(4)^2}\right) \\ &= f\left(\frac{2}{33}\right) \\ &= \frac{2}{1+2\left(\frac{2}{33}\right)^2} \\ &= \frac{2}{\left(\frac{1097}{1089}\right)} \\ &= \frac{2178}{1097}\end{aligned}$$

d. $$\begin{aligned}(g \circ g)(-1) &= g(g(-1)) \\ &= g\left(3(-1)\right) \\ &= g(-3) \\ &= 3(-3) \\ &= -9\end{aligned}$$

7. $f(x) = 2 - x \qquad g(x) = 3x+1$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(3x+1) \\ &= 2-(3x+1) \\ &= 2-3x-1 \\ &= 1-3x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(2-x) \\ &= 3(2-x)+1 \\ &= 6-3x+1 \\ &= 7-3x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f(2-x) \\ &= 2-(2-x) \\ &= 2-2+x \\ &= x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(3x+1) \\ &= 3(3x+1)+1 \\ &= 9x+3+1 \\ &= 9x+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

8. $f(x)=2x-1 \qquad g(x)=2x+1$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(2x+1)\\&=2(2x+1)-1\\&=4x+2-1\\&=4x+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(2x-1)\\&=2(2x-1)+1\\&=4x-2+1\\&=4x-1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(2x-1)\\&=2(2x-1)-1\\&=4x-2-1\\&=4x-3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g(2x+1)\\&=2(2x+1)+1\\&=4x+2+1\\&=4x+3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

9. $f(x)=3x^2+x+1 \qquad g(x)=|3x|$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(|3x|)\\&=3(|3x|)^2+(|3x|)+1\\&=27x^2+3|x|+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(3x^2+x+1\right)\\&=\left|3\left(3x^2+x+1\right)\right|\\&=3\left|3x^2+x+1\right|\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}&(f\circ f)(x)=f(f(x))\\&=f\left(3x^2+x+1\right)\\&=3\left(3x^2+x+1\right)^2+\left(3x^2+x+1\right)+1\\&=3\left(9x^4+6x^3+7x^2+2x+1\right)+3x^2+x+1+1\\&=27x^4+18x^3+24x^2+7x+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g(|3x|)\\&=|3\,|3x||\\&=9|x|\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

10. $f(x)=\sqrt{3x} \qquad g(x)=1+x+x^2$

The domain of f is $\{x \mid x\ge 0\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(1+x+x^2\right)\\&=\sqrt{3\left(1+x+x^2\right)}\\&=\sqrt{3+3x+3x^2}\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\sqrt{3x}\right)\\&=1+\sqrt{3x}+\left(\sqrt{3x}\right)^2\\&=1+\sqrt{3x}+3x\end{aligned}$$

Domain: $\{x \mid x\ge 0\}$.

$$(f\circ f)(x)=f(f(x))=f\left(\sqrt{3x}\right)=\sqrt{3\sqrt{3x}}$$

Domain: $\{x \mid x\ge 0\}$.

$$\begin{aligned}(g\circ g)(x) &= g(g(x))\\ &= g\left(1+x+x^2\right)\\ &= 1+\left(1+x+x^2\right)+\left(1+x+x^2\right)^2\\ &= 1+1+x+x^2+1+2x+3x^2+2x^3+x^4\\ &= 3+3x+4x^2+2x^3+x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

11. $f(x)=\dfrac{x+1}{x-1} \qquad g(x)=\dfrac{1}{x}$

The domain of f is $\{x \mid x\neq 1\}$.

The domain of g is $\{x \mid x\neq 0\}$.

$$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}+1}{\frac{1}{x}-1}=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-1\right)x}=\frac{1+x}{1-x}\end{aligned}$$

Domain $\{x \mid x\neq 0,\ x\neq 1\}$.

$$\begin{aligned}(g\circ f)(x) &= g(f(x))\\ &= g\left(\frac{x+1}{x-1}\right)=\frac{1}{\left(\frac{x+1}{x-1}\right)}=\frac{x-1}{x+1}\end{aligned}$$

Domain $\{x \mid x\neq -1,\ x\neq 1\}$

$$\begin{aligned}(f\circ f)(x) &= f(f(x))\\ &= f\left(\frac{x+1}{x-1}\right)=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}\\ &= \frac{\left(\frac{x+1}{x-1}+1\right)(x-1)}{\left(\frac{x+1}{x-1}-1\right)(x-1)}\\ &= \frac{x+1+x-1}{x+1-(x-1)}=\frac{2x}{2}=x\end{aligned}$$

Domain $\{x \mid x\neq 1\}$.

$$(g\circ g)(x)=g(g(x))=g\left(\frac{1}{x}\right)=\frac{1}{\left(\frac{1}{x}\right)}=x$$

Domain $\{x \mid x\neq 0\}$.

12. $f(x)=\sqrt{x-3} \qquad g(x)=\dfrac{3}{x}$

The domain of f is $\{x \mid x\geq 3\}$.

The domain of g is $\{x \mid x\neq 0\}$.

$$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{3}{x}\right)=\\ &\sqrt{\frac{3}{x}-3}\\ &= \sqrt{\frac{3-3x}{x}}\end{aligned}$$

To find the domain, we must find where $p(x)=\dfrac{3-3x}{x}>0$. p is zero or undefined when $x=1$ and $x=0$

Interval	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
Test Value	-1	$\frac{1}{2}$	2
Value of p	-6	3	$-\frac{3}{2}$
Conclusion	negative	positive	negative

Domain $\{x \mid 0<x\leq 1\}$.

$$(g\circ f)(x)=g(f(x))=g\left(\sqrt{x-3}\right)=\frac{3}{\sqrt{x-3}}$$

To find the domain, solve $x-3>0$

$x>3$

Domain $\{x \mid x>3\}$

$$(f\circ f)(x)=f(f(x))=f\left(\sqrt{x-3}\right)=\sqrt{\sqrt{x-3}-3}$$

To find the domain, solve $\sqrt{x-3}-3\geq 0$

$$\begin{aligned}\sqrt{x-3}&\geq 3\\ x-3&\geq 9\\ x&\geq 12\end{aligned}$$

Domain $\{x \mid x\geq 12\}$.

$$(g\circ g)(x)=g(g(x))=g\left(\frac{3}{x}\right)=\frac{3}{\left(\frac{3}{x}\right)}=3\left(\frac{x}{3}\right)=x$$

Domain $\{x \mid x\neq 0\}$.

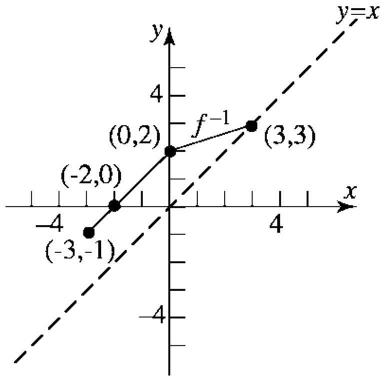
13. a. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is $\{(2,1),(5,3),(8,5),(10,6)\}$.

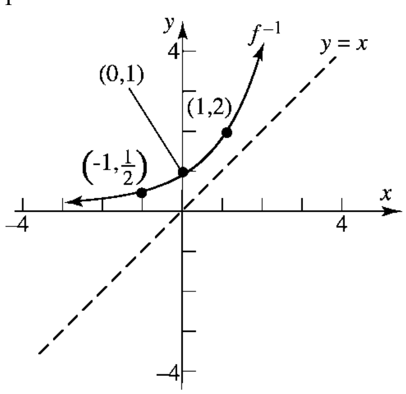
14. a. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is $\{(4,-1),(2,0),(5,1),(7,3)\}$.

15. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.



16. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.



17.
$$\begin{aligned} f(x) &= \frac{2x+3}{5x-2} \\ y &= \frac{2x+3}{5x-2} \\ x &= \frac{2y+3}{5y-2} \quad \text{Inverse} \\ x(5y-2) &= 2y+3 \\ 5xy-2x &= 2y+3 \\ 5xy-2y &= 2x+3 \\ y(5x-2) &= 2x+3 \\ y &= \frac{2x+3}{5x-2} \\ f^{-1}(x) &= \frac{2x+3}{5x-2} \end{aligned}$$

Domain of f = Range of f^{-1}

= All real numbers except $\frac{2}{5}$.

Range of f = Domain of f^{-1}

= All real numbers except $\frac{2}{5}$.

18.
$$\begin{aligned} f(x) &= \frac{2-x}{3+x} \\ y &= \frac{2-x}{3+x} \\ x &= \frac{2-y}{3+y} \quad \text{Inverse} \\ x(3+y) &= 2-y \\ 3x+xy &= 2-y \\ xy+y &= 2-3x \\ y(x+1) &= 2-3x \\ y &= \frac{2-3x}{x+1} \\ f^{-1}(x) &= \frac{2-3x}{x+1} \end{aligned}$$

Domain of f = Range of f^{-1}
= All real numbers except –3

Range of f = Domain of f^{-1}
= All real numbers except –1

19. $f(x)=\dfrac{1}{x-1}$

$y=\dfrac{1}{x-1}$

$x=\dfrac{1}{y-1}$ Inverse

$x(y-1)=1$

$xy-x=1$

$xy=x+1$

$y=\dfrac{x+1}{x}$

$f^{-1}(x)=\dfrac{x+1}{x}$

Domain of f = Range of f^{-1}
= All real numbers except 1

Range of f = Domain of f^{-1}
= All real numbers except 0

20. $f(x)=\sqrt{x-2}$

$y=\sqrt{x-2}$

$x=\sqrt{y-2}$ Inverse

$x^2=y-2 \quad x\ge 0$

$y=x^2+2 \quad x\ge 0$

$f^{-1}(x)=x^2+2 \quad x\ge 0$

Domain of f = Range of f^{-1} $=\{x\mid x\ge 2\}$

Range of f = Domain of f^{-1} $=\{x\mid x\ge 0\}$

21. $f(x)=\dfrac{3}{x^{1/3}}$

$y=\dfrac{3}{x^{1/3}}$

$x=\dfrac{3}{y^{1/3}}$ Inverse

$xy^{1/3}=3$

$y^{1/3}=\dfrac{3}{x}$

$y=\left(\dfrac{3}{x}\right)^3=\dfrac{27}{x^3}$

$f^{-1}(x)=\dfrac{27}{x^3}$

Domain of f = Range of f^{-1}
= All real numbers except 0

Range of f = Domain of f^{-1}
= All real numbers except 0

22. $f(x)=x^{1/3}+1$

$y=x^{1/3}+1$

$x=y^{1/3}+1$ Inverse

$y^{1/3}=x-1$

$y=(x-1)^3$

$f^{-1}(x)=(x-1)^3$

Domain of f = Range of f^{-1}
= All real numbers

Range of f = Domain of f^{-1}
= All real numbers

23. **a.** $f(4)=3^4=81$

b. $g(9)=\log_3(9)=\log_3\left(3^2\right)=2$

c. $f(-2)=3^{-2}=\dfrac{1}{9}$

d. $g\left(\dfrac{1}{27}\right)=\log_3\left(\dfrac{1}{27}\right)=\log_3\left(3^{-3}\right)=-3$

24. **a.** $f(1)=3^1=3$

b. $g(81)=\log_3(81)=\log_3\left(3^4\right)=4$

c. $f(-4)=3^{-4}=\dfrac{1}{81}$

d. $g\left(\dfrac{1}{243}\right)=\log_3\left(\dfrac{1}{243}\right)=\log_3\left(3^{-5}\right)=-5$

25. $5^2=z$ is equivalent to $2=\log_5 z$

26. $a^5=m$ is equivalent to $5=\log_a m$

27. $\log_5 u=13$ is equivalent to $5^{13}=u$

28. $\log_a 4=3$ is equivalent to $a^3=4$

29. $f(x)=\log(3x-2)$ requires:

$3x-2>0$

$x>\dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x>\dfrac{2}{3}\right\}$

30. $F(x) = \log_5(2x+1)$ requires:

$2x+1 > 0$

$x > -\frac{1}{2}$

Domain: $\left\{x \middle| x > -\frac{1}{2}\right\}$

31. $H(x) = \log_2\left(x^2 - 3x + 2\right)$ requires

$p(x) = x^2 - 3x + 2 > 0$

$(x-2)(x-1) > 0$

$x = 2$ and $x = 1$ are the zeros of p.

Interval	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
Test Value	0	$\frac{3}{2}$	3
Value of p	2	$-\frac{1}{4}$	2
Conclusion	positive	negative	positive

Thus, the domain of $H(x) = \log_2\left(x^2 - 3x + 2\right)$ is $\{x \mid x < 1 \text{ or } x > 2\}$.

32. $F(x) = \ln\left(x^2 - 9\right)$ requires

$p(x) = x^2 - 9 > 0$

$(x+3)(x-3) > 0$

$x = -3$ and $x = 3$ are the zeros of p.

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Test Value	–4	0	4
Value of p	7	–9	7
Conclusion	positive	negative	positive

Thus, the domain of $F(x) = \ln\left(x^2 - 9\right)$ is $\{x \mid x < -3 \text{ or } x > 3\}$.

33. $\log_2\left(\frac{1}{8}\right) = \log_2 2^{-3} = -3\log_2 2 = -3$

34. $\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$

35. $\ln e^{\sqrt{2}} = \sqrt{2}$

36. $e^{\ln 0.1} = 0.1$

37. $2^{\log_2 0.4} = 0.4$

38. $\log_2 2^{\sqrt{3}} = \sqrt{3}\log_2 2 = \sqrt{3}$

39.
$$\begin{aligned}\log_3\left(\frac{uv^2}{w}\right) &= \log_3 uv^2 - \log_3 w \\ &= \log_3 u + \log_3 v^2 - \log_3 w \\ &= \log_3 u + 2\log_3 v - \log_3 w\end{aligned}$$

40.
$$\begin{aligned}\log_2\left(a^2\sqrt{b}\right)^4 &= 4\log_2\left(a^2\sqrt{b}\right) \\ &= 4\left(\log_2 a^2 + \log_2 b^{1/2}\right) \\ &= 4\left(2\log_2 a + \frac{1}{2}\log_2 b\right) \\ &= 8\log_2 a + 2\log_2 b\end{aligned}$$

41.
$$\begin{aligned}\log\left(x^2\sqrt{x^3+1}\right) &= \log x^2 + \log\left(x^3+1\right)^{1/2} \\ &= 2\log x + \frac{1}{2}\log\left(x^3+1\right)\end{aligned}$$

42.
$$\begin{aligned}\log_5\left(\frac{x^2+2x+1}{x^2}\right) &= \log_5(x+1)^2 - \log_5\left(x^2\right) \\ &= 2\log_5(x+1) - 2\log_5 x\end{aligned}$$

43.
$$\begin{aligned}\ln\left(\frac{x\sqrt[3]{x^2+1}}{x-3}\right) &= \ln\left(x\sqrt[3]{x^2+1}\right) - \ln(x-3) \\ &= \ln x + \ln\left(x^2+1\right)^{1/3} - \ln(x-3) \\ &= \ln x + \frac{1}{3}\ln\left(x^2+1\right) - \ln(x-3)\end{aligned}$$

44.
$$\begin{aligned}&\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2 \\ &= 2\ln\left(\frac{2x+3}{x^2-3x+2}\right) \\ &= 2\left(\ln(2x+3) - \ln[(x-1)(x-2)]\right) \\ &= 2\left(\ln(2x+3) - \ln(x-1) - \ln(x-2)\right) \\ &= 2\ln(2x+3) - 2\ln(x-1) - 2\ln(x-2)\end{aligned}$$

45. $3\log_4 x^2 + \frac{1}{2}\log_4 \sqrt{x} = \log_4 \left(x^2\right)^3 + \log_4 \left(x^{1/2}\right)^{1/2}$
$$= \log_4 x^6 + \log_4 x^{1/4}$$
$$= \log_4 \left(x^6 \cdot x^{1/4}\right)$$
$$= \log_4 x^{25/4}$$
$$= \frac{25}{4}\log_4 x$$

46. $-2\log_3\left(\frac{1}{x}\right) + \frac{1}{3}\log_3 \sqrt{x}$
$$= \log_3\left(x^{-1}\right)^{-2} + \log_3\left(x^{1/2}\right)^{1/3}$$
$$= \log_3 x^2 + \log_3 x^{1/6}$$
$$= \log_3\left(x^2 \cdot x^{1/6}\right)$$
$$= \log_3 x^{13/6}$$
$$= \frac{13}{6}\log_3 x$$

47. $\ln\left(\frac{x-1}{x}\right) + \ln\left(\frac{x}{x+1}\right) - \ln\left(x^2-1\right)$
$$= \ln\left(\frac{x-1}{x} \cdot \frac{x}{x+1}\right) - \ln\left(x^2-1\right)$$
$$= \ln\left[\frac{\frac{x-1}{x+1}}{x^2-1}\right]$$
$$= \ln\left(\frac{x-1}{x+1} \cdot \frac{1}{(x-1)(x+1)}\right)$$
$$= \ln\frac{1}{(x+1)^2}$$
$$= \ln(x+1)^{-2}$$
$$= -2\ln(x+1)$$

48. $\log\left(x^2-9\right) - \log\left(x^2+7x+12\right)$
$$= \log\left(\frac{(x-3)(x+3)}{(x+3)(x+4)}\right)$$
$$= \log\left(\frac{x-3}{x+4}\right)$$

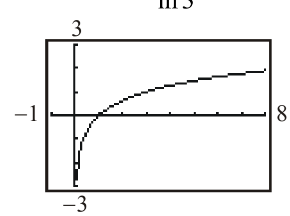
49. $2\log 2 + 3\log x - \frac{1}{2}\left[\log(x+3) + \log(x-2)\right]$
$$= \log 2^2 + \log x^3 - \frac{1}{2}\log\left[(x+3)(x-2)\right]$$
$$= \log\left(4x^3\right) - \log\left((x+3)(x-2)\right)^{1/2}$$
$$= \log\left(\frac{4x^3}{\left[(x+3)(x-2)\right]^{1/2}}\right)$$

50. $\frac{1}{2}\ln\left(x^2+1\right) - 4\ln\frac{1}{2} - \frac{1}{2}\left[\ln(x-4) + \ln x\right]$
$$= \ln\left(x^2+1\right)^{1/2} - \ln\left(\frac{1}{2}\right)^4 - \ln\left(x(x-4)\right)^{1/2}$$
$$= \ln\left(\frac{\left(x^2+1\right)^{1/2}}{\frac{1}{16}\left[x(x-4)\right]^{1/2}}\right)$$
$$= \ln\left(\frac{16\sqrt{x^2+1}}{\sqrt{x(x-4)}}\right)$$

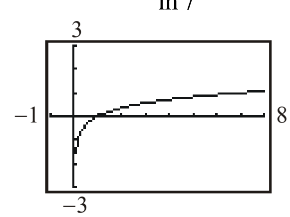
51. $\log_4 19 = \frac{\log 19}{\log 4} \approx 2.124$

52. $\log_2 21 = \frac{\log 21}{\log 2} \approx 4.392$

53. $Y_1 = \log_3 x = \frac{\ln x}{\ln 3}$

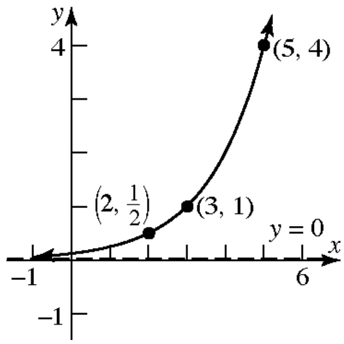


54. $Y_1 = \log_7 x = \frac{\ln x}{\ln 7}$



55. $f(x) = 2^{x-3}$

Using the graph of $y = 2^x$, shift the graph 3 units to the right.



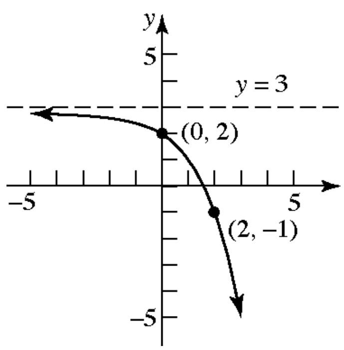
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$

56. $f(x) = -2^x + 3$

Using the graph of $y = 2^x$, reflect the graph about the x-axis, and shift vertically 3 units up.



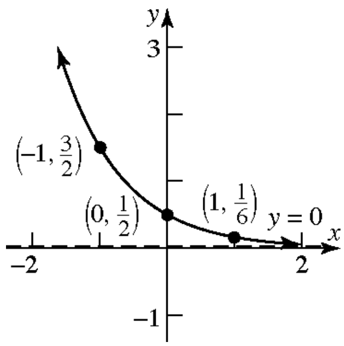
Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

Horizontal Asymptote: $y = 3$

57. $f(x) = \frac{1}{2}\left(3^{-x}\right)$

Using the graph of $y = 3^x$, reflect the graph about the y-axis, and shrink vertically by a factor of $\frac{1}{2}$.



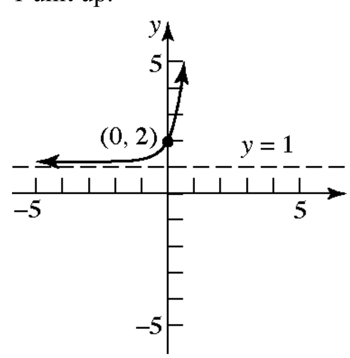
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$

58. $f(x) = 1 + 3^{2x}$

Using the graph of $y = 3^x$, shrink the graph horizontally by a factor of $\frac{1}{2}$, and shift vertically 1 unit up.



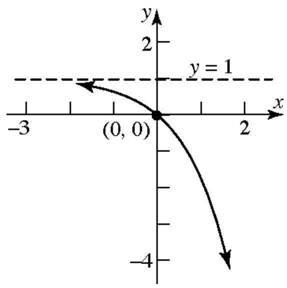
Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

Horizontal Asymptote: $y = 1$

59. $f(x) = 1 - e^x$

Using the graph of $y = e^x$, reflect about the x-axis, and shift up 1 unit.



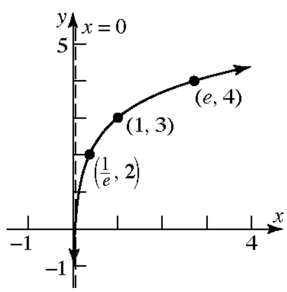
Domain: $(-\infty, \infty)$

Range: $(-\infty, 1)$

Horizontal Asymptote: $y = 1$

60. $f(x) = 3 + \ln x$

Using the graph of $y = \ln x$, shift up 3 units.



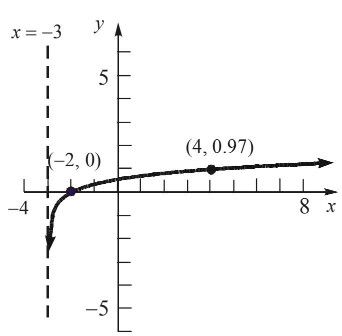
Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$

61. $f(x) = \frac{1}{2}\ln(x+3)$

Using the graph of $y = \ln x$, shift left 3 units and shrink vertically by a factor of $\frac{1}{2}$.



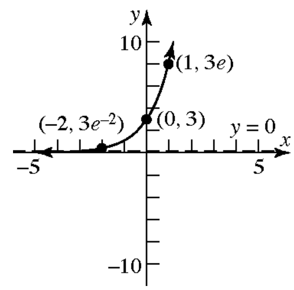
Domain: $(-3, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = -3$

62. $f(x) = 3e^x$

Using the graph of $y = e^x$, stretch vertically by a factor of 3.



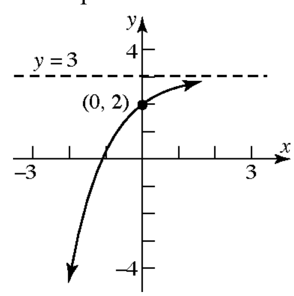
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$

63. $f(x) = 3 - e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 3 units.



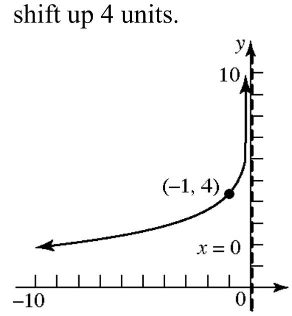
Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

Horizontal Asymptote: $y = 3$

64. $f(x) = 4 - \ln(-x)$

Using the graph of $y = \ln x$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 4 units.



Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = 0$

65.
$$\begin{aligned}4^{1-2x} &= 2\\ \left(2^2\right)^{1-2x} &= 2\\ 2^{2-4x} &= 2^1\\ 2-4x &= 1\\ -4x &= -1\\ x &= \frac{1}{4}\end{aligned}$$
The solution set is $\left\{\frac{1}{4}\right\}$.

66.
$$\begin{aligned}8^{6+3x} &= 4\\ \left(2^3\right)^{6+3x} &= 2^2\\ 2^{18+9x} &= 2^2\\ 18+9x &= 2\\ 9x &= -16\\ x &= -\frac{16}{9}\end{aligned}$$
The solution set is $\left\{-\frac{16}{9}\right\}$.

67.
$$\begin{aligned}3^{x^2+x} &= \sqrt{3}\\ 3^{x^2+x} &= 3^{1/2}\\ x^2+x &= \frac{1}{2}\\ 2x^2+2x-1 &= 0\end{aligned}$$
$$\begin{aligned}x &= \frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}\\ &= \frac{-2\pm\sqrt{12}}{4}\\ &= \frac{-2\pm2\sqrt{3}}{4}\\ &= \frac{-1\pm\sqrt{3}}{2}\end{aligned}$$
The solution set is $\left\{\frac{-1-\sqrt{3}}{2},\ \frac{-1+\sqrt{3}}{2}\right\}$.

68.
$$\begin{aligned}4^{x-x^2} &= \frac{1}{2}\\ 2^{2\left(x-x^2\right)} &= 2^{-1}\\ 2x-2x^2 &= -1\\ 2x^2-2x-1 &= 0\end{aligned}$$
$$\begin{aligned}x &= \frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-1)}}{2(2)}\\ &= \frac{2\pm\sqrt{12}}{4}\\ &= \frac{2\pm2\sqrt{3}}{4}\\ &= \frac{1\pm\sqrt{3}}{2}\end{aligned}$$
The solution set is $\left\{\frac{1-\sqrt{3}}{2},\ \frac{1+\sqrt{3}}{2}\right\}$.

69.
$$\begin{aligned}\log_x 64 &= -3\\ x^{-3} &= 64\\ \left(x^{-3}\right)^{-1/3} &= 64^{-1/3}\\ x &= \frac{1}{\sqrt[3]{64}} = \frac{1}{4}\end{aligned}$$
The solution set is $\left\{\frac{1}{4}\right\}$.

70.
$$\begin{aligned}\log_{\sqrt{2}} x &= -6\\ x &= \left(\sqrt{2}\right)^{-6}\\ &= \left(2^{1/2}\right)^{-6}\\ &= 2^{-3} = \frac{1}{8}\end{aligned}$$
The solution set is $\left\{\frac{1}{8}\right\}$.

71.
$$5^x = 3^{x+2}$$
$$\log\left(5^x\right) = \log\left(3^{x+2}\right)$$
$$x\log 5 = (x+2)\log 3$$
$$x\log 5 = x\log 3 + 2\log 3$$
$$x\log 5 - x\log 3 = 2\log 3$$
$$x(\log 5 - \log 3) = 2\log 3$$
$$x = \frac{2\log 3}{\log 5 - \log 3} \approx 4.301$$
The solution set is $\left\{\frac{2\log 3}{\log 5 - \log 3} \approx 4.301\right\}$.

72.
$$5^{x+2} = 7^{x-2}$$
$$\log\left(5^{x+2}\right) = \log\left(7^{x-2}\right)$$
$$(x+2)\log 5 = (x-2)\log 7$$
$$x\log 5 + 2\log 5 = x\log 7 - 2\log 7$$
$$x\log 5 - x\log 7 = -2\log 7 - 2\log 5$$
$$x(\log 5 - \log 7) = -2\log 7 - 2\log 5$$
$$x = \frac{-2\log 7 - 2\log 5}{\log 5 - \log 7} \approx 21.133$$
The solution set is $\left\{\frac{-2\log 7 - 2\log 5}{\log 5 - \log 7} \approx 21.133\right\}$.

73.
$$9^{2x} = 27^{3x-4}$$
$$\left(3^2\right)^{2x} = \left(3^3\right)^{3x-4}$$
$$3^{4x} = 3^{9x-12}$$
$$4x = 9x - 12$$
$$-5x = -12$$
$$x = \frac{12}{5}$$
The solution set is $\left\{\frac{12}{5}\right\}$.

74.
$$25^{2x} = 5^{x^2-12}$$
$$\left(5^2\right)^{2x} = 5^{x^2-12}$$
$$5^{4x} = 5^{x^2-12}$$
$$4x = x^2 - 12$$
$$x^2 - 4x - 12 = 0$$
$$(x-6)(x+2) = 0$$
$$x = 6 \text{ or } x = -2$$
The solution set is $\{-2, 6\}$.

75.
$$\log_3 \sqrt{x-2} = 2$$
$$\sqrt{x-2} = 3^2$$
$$\sqrt{x-2} = 9$$
$$x - 2 = 9^2$$
$$x - 2 = 81$$
$$x = 83$$
Check: $\log_3 \sqrt{83-2} = \log_3 \sqrt{81}$
$= \log_3 9$
$= 2$

The solution set is $\{83\}$.

76.
$$2^{x+1} \cdot 8^{-x} = 4$$
$$2^{x+1} \cdot \left(2^3\right)^{-x} = 2^2$$
$$2^{x+1} \cdot 2^{-3x} = 2^2$$
$$2^{-2x+1} = 2^2$$
$$-2x + 1 = 2$$
$$-2x = 1$$
$$x = -\frac{1}{2}$$
The solution set is $\left\{-\frac{1}{2}\right\}$.

77.
$$8 = 4^{x^2} \cdot 2^{5x}$$
$$2^3 = \left(2^2\right)^{x^2} \cdot 2^{5x}$$
$$2^3 = 2^{2x^2+5x}$$
$$3 = 2x^2 + 5x$$
$$0 = 2x^2 + 5x - 3$$
$$0 = (2x-1)(x+3)$$
$$x = \frac{1}{2} \text{ or } x = -3$$
The solution set is $\left\{-3, \frac{1}{2}\right\}$.

78. $2^x \cdot 5 = 10^x$

$\ln\left(2^x \cdot 5\right) = \ln 10^x$

$\ln 2^x + \ln 5 = \ln 10^x$

$x\ln 2 + \ln 5 = x\ln 10$

$\ln 5 = x\ln 10 - x\ln 2$

$\ln 5 = x(\ln 10 - \ln 2)$

$\dfrac{\ln 5}{\ln 10 - \ln 2} = x$

$x = \dfrac{\ln 5}{\ln \frac{10}{2}} = \dfrac{\ln 5}{\ln 5} = 1$

The solution set is $\{1\}$.

79. $\log_6(x+3) + \log_6(x+4) = 1$

$\log_6\left((x+3)(x+4)\right) = 1$

$(x+3)(x+4) = 6^1$

$x^2 + 7x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -6$ or $x = -1$

Since $\log_6(-6+3) = \log_6(-3)$ is undefined, the solution set is $\{-1\}$.

80. $\log(7x-12) = 2\log x$

$\log(7x-12) = \log x^2$

$7x - 12 = x^2$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 4$ or $x = 3$

Since each original logarithm is defined, for $x = 3$ and $x = 4$, the solution set is $\{3, 4\}$.

81. $e^{1-x} = 5$

$1 - x = \ln 5$

$-x = -1 + \ln 5$

$x = 1 - \ln 5 \approx -0.609$

The solution set is $\{1 - \ln 5 \approx -0.609\}$.

82. $e^{1-2x} = 4$

$1 - 2x = \ln 4$

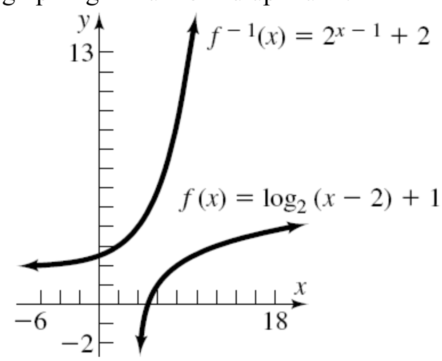
$-2x = -1 + \ln 4$

$x = \dfrac{1 - \ln 4}{2} \approx -0.193$

The solution set is $\left\{\dfrac{1-\ln 4}{2} \approx -0.193\right\}$.

83. **a.** $f(x) = \log_2(x-2) + 1$

Using the graph of $y = \log_2 x$, shift the graph right 2 units and up 1 unit.



b. $f(6) = \log_2(6-2) + 1$

$= \log_2(4) + 1$

$= 2 + 1$

$= 3$

The point $(6, 3)$ is on the graph of *f*.

c. $f(x) = 4$

$\log_2(x-2) + 1 = 4$

$\log_2(x-2) = 3$

$x - 2 = 2^3$

$x - 2 = 8$

$x = 10$

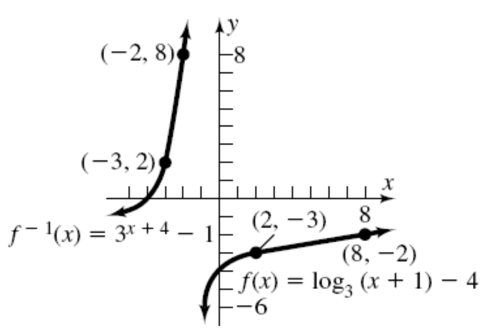
The point $(10, 4)$ is on the graph of *f*.

d.
$$\begin{aligned} f(x) &= 0 \\ \log_2(x-2)+1 &= 0 \\ \log_2(x-2) &= -1 \\ x-2 &= 2^{-1} \\ x-2 &= \frac{1}{2} \\ x &= \frac{5}{2} \end{aligned}$$
The zero of *f* is $\frac{5}{2}$.

e. Based on parts (a) and (d), $f(x) > 0$ when $x > \frac{5}{2}$. The solution set is $\left\{x \mid x > \frac{5}{2}\right\}$ or $\left(\frac{5}{2}, \infty\right)$.

f.
$$\begin{aligned} y &= \log_2(x-2)+1 \\ x &= \log_2(y-2)+1 \\ x-1 &= \log_2(y-2) \\ 2^{x-1} &= y-2 \\ 2^{x-1}+2 &= y \quad \text{(inverse)} \\ f^{-1}(x) &= 2^{x-1}+2 \end{aligned}$$
[The inverse is graphed in part (a)]

84. a. $f(x) = \log_3(x+1) - 4$
Using the graph of $\log_3 x$, shift the graph left 1 unit and down 4 units.



b.
$$\begin{aligned} f(8) &= \log_3(8+1) - 4 \\ &= \log_3(9) - 4 \\ &= 2 - 4 \\ &= -2 \end{aligned}$$
The point $(8, -2)$ is on the graph of *f*.

c.
$$\begin{aligned} f(x) &= -3 \\ \log_3(x+1) - 4 &= -3 \\ \log_3(x+1) &= 1 \\ x+1 &= 3^1 \\ x+1 &= 3 \\ x &= 2 \end{aligned}$$
The point $(2, -3)$ is on the graph of *f*.

d.
$$\begin{aligned} f(x) &= 0 \\ \log_3(x+1) - 4 &= 0 \\ \log_3(x+1) &= 4 \\ x+1 &= 3^4 \\ x+1 &= 81 \\ x &= 80 \end{aligned}$$
The zero of *f* is 80.

e. Using parts (a) and (d), we see that $f(x) < 0$ for $-1 < x < 80$. The solution set is $\{x \mid -1 < x < 80\}$ or $(-1, 80)$.

f.
$$\begin{aligned} y &= \log_3(x+1) - 4 \\ x &= \log_3(y+1) - 4 \\ x+4 &= \log_3(y+1) \\ 3^{x+4} &= y+1 \\ 3^{x+4} - 1 &= y \quad \text{(inverse)} \\ f^{-1}(x) &= 3^{x+4} - 1 \end{aligned}$$
[The inverse is graphed in part (a)]

85.
$$\begin{aligned} h(300) &= \left(30(0)+8000\right)\log\left(\frac{760}{300}\right) \\ &\approx 3229.5 \text{ meters} \end{aligned}$$

86.
$$\begin{aligned} h(500) &= \left(30(5)+8000\right)\log\left(\frac{760}{500}\right) \\ &\approx 1482 \text{ meters} \end{aligned}$$

87. $P = 25e^{0.1d}$

a. $P = 25e^{0.1(4)} = 25e^{0.4} \approx 37.3$ watts

b.
$$\begin{aligned} 50 &= 25e^{0.1d} \\ 2 &= e^{0.1d} \\ \ln 2 &= 0.1d \\ d &= \frac{\ln 2}{0.1} \approx 6.9 \text{ decibels} \end{aligned}$$

88. $L = 9 + (5.1)\log d$

a. $L = 9 + (5.1)\log 3.5 \approx 11.77$

b.
$$\begin{aligned} 14 &= 9 + (5.1)\log d \\ 5 &= (5.1)\log d \\ \log d &= \frac{5}{5.1} \approx 0.9804 \\ d &\approx 10^{0.9804} \approx 9.56 \text{ inches} \end{aligned}$$

89. a. $n = \dfrac{\log 10{,}000 - \log 90{,}000}{\log(1-0.20)} \approx 9.85$ years

b.
$$\begin{aligned} n &= \frac{\log(0.5i) - \log(i)}{\log(1-0.15)} \\ &= \frac{\log\left(\frac{0.5i}{i}\right)}{\log 0.85} \\ &= \frac{\log 0.5}{\log 0.85} \\ &\approx 4.27 \text{ years} \end{aligned}$$

90. In 18 years,
$$\begin{aligned} A &= 10{,}000\left(1 + \frac{0.04}{2}\right)^{(2)(18)} \\ &= 10{,}000(1.02)^{36} \\ &\approx \$20{,}398.87 \end{aligned}$$

The effective interest rate is computed as follows:

When $t = 1$,
$$\begin{aligned} A &= 10{,}000\left(1 + \frac{0.04}{2}\right)^{(2)(1)} \\ &= 10{,}000(1.02)^{2} \\ &= \$10{,}404 \end{aligned}$$

Note, $\dfrac{10{,}404 - 10{,}000}{10{,}000} = \dfrac{404}{10{,}000} = 0.0404$, so the effective interest rate is 4.04%.

In order for the bond to double in value, we have the equation: $A = 2P$.

$$\begin{aligned} 10{,}000\left(1 + \frac{0.04}{2}\right)^{2t} &= 20{,}000 \\ (1.02)^{2t} &= 2 \\ 2t \ln 1.02 &= \ln 2 \\ t &= \frac{\ln 2}{2\ln 1.02} \approx 17.5 \text{ years} \end{aligned}$$

91.
$$\begin{aligned} P &= A\left(1 + \frac{r}{n}\right)^{-nt} \\ &= 85{,}000\left(1 + \frac{0.04}{2}\right)^{-2(18)} \\ &\approx \$41{,}668.97 \end{aligned}$$

92. a.
$$\begin{aligned} 5000 &= 620.17e^{r(20)} \\ \frac{5000}{620.17} &\approx e^{20r} \\ \ln\left(\frac{5000}{620.17}\right) &\approx 20r \\ r &\approx \frac{\ln\left(\frac{5000}{620.17}\right)}{20} \approx 0.10436 \\ r &\approx 10.436\% \end{aligned}$$

b. $A = 4000e^{0.10436(20)} \approx \$32{,}249.24$

The bank's claim is correct.

93.
$$\begin{aligned} A &= A_0 e^{kt} \\ 0.5A_0 &= A_0 e^{k(5600)} \\ 0.5 &= e^{5600k} \\ \ln 0.5 &= 5600k \\ k &= \frac{\ln 0.5}{5600} \\ 0.05A_0 &= A_0 e^{\left(\frac{\ln 0.5}{5600}\right)t} \\ 0.05 &= e^{\left(\frac{\ln 0.5}{5600}\right)t} \\ \ln 0.05 &= \left(\frac{\ln 0.5}{5600}\right)t \\ t &= \frac{\ln 0.05}{\left(\frac{\ln 0.5}{5600}\right)} \approx 24{,}203 \end{aligned}$$

The man died approximately 24,203 years ago.

94. Using $u = T + (u_0 - T)e^{kt}$, with $t = 5$, $T = 70$, $u_0 = 450$, and $u = 400$.

$$400 = 70 + (450 - 70)e^{k(5)}$$
$$330 = 380e^{5k}$$
$$\frac{330}{380} = e^{5k}$$
$$\ln\left(\frac{330}{380}\right) = 5k$$
$$k = \frac{\ln(330/380)}{5}$$

Find time for temperature of 150˚F:

$$150 = 70 + (450 - 70)e^{\left(\frac{\ln(330/380)}{5}\right)t}$$
$$80 = 380e^{\left(\frac{\ln(330/380)}{5}\right)t}$$
$$\frac{80}{380} = e^{\left(\frac{\ln(330/380)}{5}\right)t}$$
$$\ln\left(\frac{80}{380}\right) = \left(\frac{\ln(330/380)}{5}\right)t$$
$$t = \frac{\ln\left(\frac{80}{380}\right)}{\frac{\ln(330/380)}{5}} \approx 55.22$$

The temperature of the skillet will be 150˚F after approximately 55.22 minutes (or 55 minutes, 13 seconds).

95. $P = P_0e^{kt}$

$$= 6,302,486,693e^{0.0167(7)}$$
$$\approx 6,835,600,129 \text{ people}$$

96.
$$A = A_0e^{kt}$$
$$0.5A_0 = A_0e^{k(5.27)}$$
$$0.5 = e^{5.27k}$$
$$\ln 0.5 = 5.27k$$
$$k = \frac{\ln 0.5}{5.27}$$

In 20 years: $A = 100e^{\left(\frac{\ln 0.5}{5.27}\right)(20)} \approx 7.204$ grams

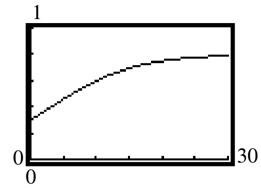
In 40 years: $A = 100e^{\left(\frac{\ln 0.5}{5.27}\right)(40)} \approx 0.519$ grams

97. a. $P(0) = \dfrac{0.8}{1+1.67e^{-0.16(0)}} = \dfrac{0.8}{1+1.67} \approx 0.3$

In 2003, about 30% of cars had a GPS.

b. The maximum proportion is the carrying capacity, $c = 0.8 = 80\%$.

c. $Y_1 = \dfrac{0.8}{1+1.67e^{-0.16x}}$

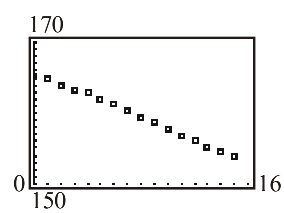


d. Find t such that $P(t) = 0.75$.

$$\frac{0.8}{1+1.67e^{-0.16t}} = 0.75$$
$$0.8 = 0.75\left(1+1.67e^{-0.16t}\right)$$
$$\frac{0.8}{0.75} = 1 + 1.67e^{-0.16t}$$
$$\frac{0.8}{0.75} - 1 = 1.67e^{-0.16t}$$
$$\frac{\frac{0.8}{0.75} - 1}{1.67} = e^{-0.16t}$$
$$\ln\left(\frac{\frac{0.8}{0.75} - 1}{1.67}\right) = -0.16t$$
$$t = \frac{\ln\left(\frac{\frac{0.8}{0.75} - 1}{1.67}\right)}{-0.16} \approx 20.13$$

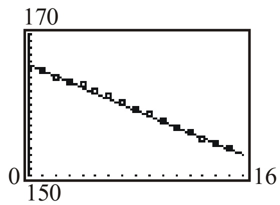
Note that $2003 + 20.13 = 2023.13$, so 75% of new cars will have GPS in 2023.

98. a.



b. Using EXPonential REGression on the data yields: $y = (165.73)(0.9951)^x$

c. $Y_1 = (165.73)(0.9951)^x$

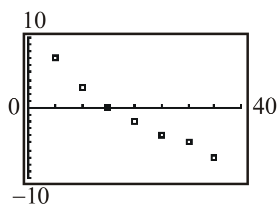


d. Find x when $y = 110$.

$$(165.73)(0.9951)^x = 110$$
$$(0.9951)^x = \frac{110}{165.73}$$
$$x \ln 0.9951 = \ln\left(\frac{110}{165.73}\right)$$
$$x = \frac{\ln\left(\frac{110}{165.73}\right)}{\ln 0.9951} \approx 83$$

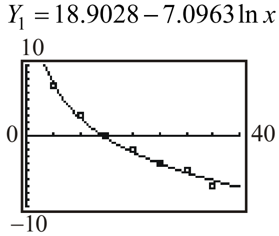
Therefore, it will take approximately 83 seconds for the probe to reach a temperature of 110˚F.

99. a.



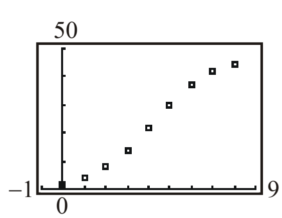
b. Using LnREGression on the data yields: $y = 18.9028 - 7.0963 \ln x$

c. $Y_1 = 18.9028 - 7.0963 \ln x$



d. If $x = 23$, then $y = 18.9028 - 7.0963 \ln 23 \approx -3^{\circ}\text{F}$.

100. a.

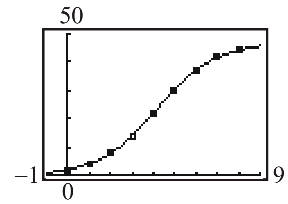


The data appear to have a logistic relation

b. Using LOGISTIC REGression on the data yields:

$$C = \frac{46.93}{1 + 21.273e^{-0.7306t}}$$

c. $Y_1 = \dfrac{46.93}{1 + 21.273e^{-0.7306t}}$:



d. As $t \to \infty$, $21.2733e^{-0.7306t} \to 0$, which means $1 + 21.2733e^{-0.7306t} \to 1$, so

$$C = \frac{46.9292}{1 + 21.2733e^{-0.7306t}} \to 46.9292$$

Therefore, according to the function, a maximum of about 47 people can catch the cold.

In reality, all 50 people living in the town might catch the cold.

e. Find t when $C = 10$.

$$\frac{46.9292}{1 + 21.2733e^{-0.7306t}} = 10$$
$$46.9292 = 10\left(1 + 21.2733e^{-0.7306t}\right)$$
$$\frac{46.9292}{10} = 1 + 21.2733e^{-0.7306t}$$
$$\frac{46.9292}{10} - 1 = 21.2733e^{-0.7306t}$$
$$3.69292 = 21.2733e^{-0.7306t}$$
$$\frac{3.69292}{21.2733} = e^{-0.7306t}$$
$$\ln\left(\frac{3.69292}{21.2733}\right) = -0.7306t$$
$$\frac{\ln\left(\frac{3.69292}{21.2733}\right)}{-0.7306} = t$$
$$t \approx 2.4$$

Therefore, after approximately 2.4 days (during the 10th hour on the 3rd day), 10 people had caught the cold.

f. Find t when $C = 46$.

$$\frac{46.9292}{1+21.2733e^{-0.7306t}} = 46$$
$$46.9292 = 46\left(1+21.2733e^{-0.7306t}\right)$$
$$\frac{46.9292}{46} = 1+21.2733e^{-0.7306t}$$
$$\frac{46.9292}{46} - 1 = 21.2733e^{-0.7306t}$$
$$0.0202 = 21.2733e^{-0.7306t}$$
$$\frac{0.0202}{21.2733} = e^{-0.7306t}$$
$$\frac{0.0202}{21.2733} = e^{-0.7306t}$$
$$\ln\left(\frac{0.0202}{21.2733}\right) = -0.7306t$$
$$\frac{\ln\left(\frac{0.0202}{21.2733}\right)}{-0.7306} = t$$
$$t \approx 9.5$$

Therefore, after approximately 9.5 days (during the 12th hour on the 10th day), 46 people had caught the cold.

Chapter 4 Test

1. $f(x) = \dfrac{x+2}{x-2}$ $\qquad g(x) = 2x+5$

The domain of f is $\{x \mid x \neq 2\}$.

The domain of g is all real numbers.

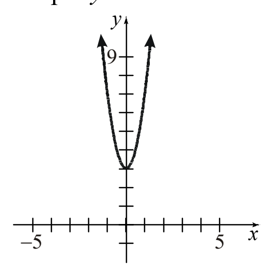
a.
$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x+5) \\ &= \frac{(2x+5)+2}{(2x+5)-2} \\ &= \frac{2x+7}{2x+3}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \neq -\dfrac{3}{2}\right\}$.

b.
$$\begin{aligned}(g \circ f)(x) &= g(f(-2)) \\ &= g\left(\frac{-2+2}{-2-2}\right) \\ &= g(0) \\ &= 2(0)+5 \\ &= 5\end{aligned}$$

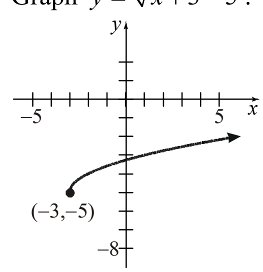
c.
$$\begin{aligned}(f \circ g)(x) &= f(g(-2)) \\ &= f(2(-2)+5) \\ &= f(1) \\ &= \frac{1+2}{1-2} \\ &= \frac{3}{-1} \\ &= -3\end{aligned}$$

2. a. Graph $y = 4x^2 + 3$:



The function is not one-to-one because it fails the horizontal line test. A horizontal line (for example, $y = 4$) intersects the graph twice.

b. Graph $y = \sqrt{x+3} - 5$:



The function is one-to-one because it passes the horizontal line test. Every horizontal line intersects the graph at most once.

3. $$f(x) = \frac{2}{3x-5}$$
$$y = \frac{2}{3x-5}$$
$$x = \frac{2}{3y-5} \quad \text{Inverse}$$
$$x(3y-5) = 2$$
$$3xy - 5x = 2$$
$$3xy = 5x + 2$$
$$y = \frac{5x+2}{3x}$$
$$f^{-1}(x) = \frac{5x+2}{3x}$$

Domain of f = Range of f^{-1}

= All real numbers except $\frac{5}{3}$.

Range of f = Domain of f^{-1}

= All real numbers except 0.

4. If the point $(3, -5)$ is on the graph of f, then the point $(-5, 3)$ must be on the graph of f^{-1}.

5. $3^x = 243$
$3^x = 3^5$
$x = 5$

6. $\log_b 16 = 2$
$b^2 = 16$
$b = \pm\sqrt{16} = \pm 4$
Since the base of a logarithm must be positive, the only viable solution is $b = 4$.

7. $\log_5 x = 4$
$x = 5^4$
$x = 625$

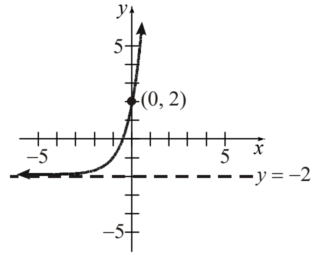
8. $e^3 + 2 \approx 22.086$

9. $\log 20 \approx 1.301$

10. $\log_3 21 = \frac{\log 21}{\log 3} \approx 2.771$

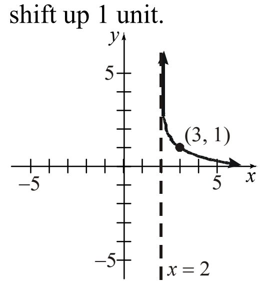
11. $\ln 133 \approx 4.890$

12. $f(x) = 4^{x+1} - 2$
Using the graph of $y = 4^x$, shift the graph to the left 1 unit and down 2 units.



Domain: $(-\infty, \infty)$
Range: $(-2, \infty)$
Horizontal Asymptote: $y = -2$

13. $g(x) = 1 - \log_5(x-2)$
Using the graph of $y = \log_5 x$, shift the graph to the right 2 units, reflect about the x-axis, and shift up 1 unit.



Domain: $(2, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 2$

14. $5^{x+2} = 125$
$5^{x+2} = 5^3$
$x + 2 = 3$
$x = 1$
The solution set is $\{1\}$.

15. $\log(x+9) = 2$
$x + 9 = 10^2$
$x + 9 = 100$
$x = 91$
The solution set is $\{91\}$.

16. $8-2e^{-x}=4$

$$\begin{aligned}-2e^{-x}&=-4\\ e^{-x}&=2\\ -x&=\ln 2\\ x&=-\ln 2\approx -0.693\end{aligned}$$

The solution set is $\{-\ln 2\approx -0.693\}$.

17. $\log\left(x^2+3\right)=\log\left(x+6\right)$

$$\begin{aligned}x^2+3&=x+6\\ x^2-x-3&=0\\ x&=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}\\ &=\frac{1\pm\sqrt{13}}{2}\end{aligned}$$

The solution set is $\left\{\frac{1-\sqrt{13}}{2},\ \frac{1+\sqrt{13}}{2}\right\}$.

18.

$$\begin{aligned}7^{x+3}&=e^x\\ \ln 7^{x+3}&=\ln e^x\\ (x+3)\ln 7&=x\\ x\ln 7+3\ln 7&=x\\ x\ln 7-x&=-3\ln 7\\ x(\ln 7-1)&=-3\ln 7\\ x&=\frac{-3\ln 7}{\ln 7-1}=\frac{3\ln 7}{1-\ln 7}\approx -6.172\end{aligned}$$

The solution set is $\left\{\frac{3\ln 7}{1-\ln 7}\approx -6.172\right\}$.

19. $\log_2\left(x-4\right)+\log_2\left(x+4\right)=3$

$$\begin{aligned}\log_2\left[\left(x-4\right)\left(x+4\right)\right]&=3\\ \log_2\left(x^2-16\right)&=3\\ x^2-16&=2^3\\ x^2-16&=8\\ x^2&=24\\ x&=\pm\sqrt{24}=\pm 2\sqrt{6}\end{aligned}$$

Because $x=-2\sqrt{6}$ results in a negative arguments for the original logarithms, the only viable solution is $x=2\sqrt{6}$. That is, the solution set is $\left\{2\sqrt{6}\right\}$.

20. $\log_2\left(\frac{4x^3}{x^2-3x-18}\right)$

$$\begin{aligned}&=\log_2\left(\frac{2^2x^3}{(x+3)(x-6)}\right)\\ &=\log_2\left(2^2x^3\right)-\log_2\left[(x+3)(x-6)\right]\\ &=\log_2 2^2+\log_2 x^3-\left[\log_2(x+3)+\log_2(x-6)\right]\\ &=2+3\log_2 x-\log_2(x+3)-\log_2(x-6)\end{aligned}$$

21.

$$\begin{aligned}A&=A_0e^{kt}\\ 34&=50e^{k(30)}\\ 0.68&=e^{30k}\\ \ln 0.68&=30k\\ k&=\frac{\ln 0.68}{30}\end{aligned}$$

Thus, the decay model is $A=50e^{\left(\frac{\ln 0.68}{30}\right)t}$.
We need to find t when $A=2$:

$$\begin{aligned}2&=50e^{\left(\frac{\ln 0.68}{30}\right)t}\\ 0.04&=e^{\left(\frac{\ln 0.68}{30}\right)t}\\ \ln 0.04&=\left(\frac{\ln 0.68}{30}\right)t\\ t&=\frac{\ln 0.04}{\left(\frac{\ln 0.68}{30}\right)}\approx 250.39\end{aligned}$$

There will be 2 mg of the substance remaining after about 250 days.

22. **a.** The 2013 – 2014 academic year is $t=10$ years after the 2003 – 2004 academic year.

We use the formula $A=P\left(1+\frac{r}{n}\right)^{nt}$, with $P=19{,}710$, $r=0.06$, $n=1$, and $t=10$.

$$\begin{aligned}A&=19{,}710\left(1+\frac{0.06}{1}\right)^{(1)(10)}\\ &=19{,}710(1.06)^{10}\\ &\approx \$35{,}297.61\end{aligned}$$

In 2013 – 2014, the average cost of college at 4-year private colleges will be about $35,297.61.

b. We use the formula $A = Pe^{rt}$, with $A = 35,297.61$, $r = 0.06$, and $t = 10$.

$$35,297.61 = Pe^{0.05(10)}$$
$$35,297.61 = Pe^{0.5}$$
$$P = \frac{35,297.61}{e^{0.5}} \approx \$21,409.08$$

About \$21,409.08 will need to be put into the savings plan.

23. a.
$$80 = 10\log\left(\frac{I}{10^{-12}}\right)$$
$$8 = \log\left(\frac{I}{10^{-12}}\right)$$
$$8 = \log I - \log 10^{-12}$$
$$8 = \log I - (-12)$$
$$8 = \log I + 12$$
$$-4 = \log I$$
$$I = 10^{-4} = 0.0001$$

If one person shouts, the intensity is 10^{-4} watts per square meter. Thus, if two people shout at the same time, the intensity will be 2×10^{-4} watts per square meter. Thus, the loudness will be

$$D = 10\log\left(\frac{2\times 10^{-4}}{10^{-12}}\right) = 10\log\left(2\times 10^{8}\right) \approx 83.01$$

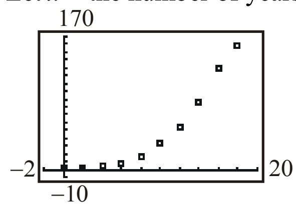
decibels

b. Let n represent the number of people who must shout. Then the intensity will be $n\times 10^{-4}$. If $D = 125$, then

$$125 = 10\log\left(\frac{n\times 10^{-4}}{10^{-12}}\right)$$
$$125 = 10\log\left(n\times 10^{8}\right)$$
$$12.5 = \log\left(n\times 10^{8}\right)$$
$$n\times 10^{8} = 10^{12.5}$$
$$n = 10^{4.5} \approx 31,623$$

About 31,623 people would have to shout at the same time in order for the resulting sound level to meet the pain threshold.

24. Let x = the number of years since 1985.



Using LOGISTIC REGression on the data yields:

$$y = \frac{213.005}{1 + 205.859e^{-0.3564x}}$$

Note that $2007 - 1985 = 22$. Substituting $x = 22$, we find that

$$y = \frac{213.005}{1 + 205.859e^{-0.3564(22)}} \approx 197.05.$$

Thus, we predict that in 2007 there will be about 197.05 million U.S. cell phone subscribers.

Chapter 4 Projects

Project 1

a. Newton's Law of Cooling:

$u(t) = T + (u_0 - T)e^{kt}$, k < 0

Container 1: u_0 = 200ºF, T = 70ºF, u(30)=100ºF, t = 30 mins.

$$100 = 70 + (200 - 70)e^{30k}$$
$$30 = 130e^{30k}$$
$$\frac{30}{130} = e^{30k}$$
$$30k = \ln\left(\frac{30}{130}\right)$$
$$k = \frac{\ln\left(\frac{30}{130}\right)}{30} \approx -0.04888$$
$$u_1(t) = 70 + 130e^{-0.04888t}$$

Container 2: $u_0 = 200°F$, T = 60ºF, u(25)=110ºF, t = 25 mins.

$$100 = 60 + (200 - 60)e^{25k}$$
$$50 = 140e^{25k}$$
$$\frac{50}{140} = e^{25k}$$
$$25k = \ln\left(\frac{50}{140}\right)$$
$$k = \frac{\ln\left(\frac{50}{140}\right)}{25} \approx -0.04118$$
$$u_2(t) = 60 + 140e^{-0.04118t}$$

Container 3: $u_0 = 200°F$, T = 65ºF, u(20)=120ºF, t = 20 mins.

$$100 = 65 + (200 - 65)e^{20k}$$
$$55 = 135e^{20k}$$
$$\frac{55}{135} = e^{20k}$$
$$20k = \ln\left(\frac{55}{135}\right)$$
$$k = \frac{\ln\left(\frac{55}{135}\right)}{20} \approx -0.04490$$
$$u_3(t) = 65 + 135e^{-0.04490t}$$

b. We need time for each of the problems, so solve for t first then substitute the specific values for each container:

$$u = T + (u_0 - T)e^{kt}$$
$$u - T = (u_0 - T)e^{kt}$$
$$\frac{u - T}{u_0 - T} = e^{kt}$$
$$kt = \ln\left(\frac{u - T}{u_0 - T}\right)$$
$$t = \frac{\ln\left(\frac{u - T}{u_0 - T}\right)}{k}$$

Container 1:

$$t = \frac{\ln\left(\frac{130 - 70}{200 - 70}\right)}{-0.04888} \approx 15.82 \text{ minutes}$$

Container 2:

$$t = \frac{\ln\left(\frac{130 - 60}{200 - 60}\right)}{-0.04118} \approx 16.83 \text{ minutes}$$

Container 3:

$$t = \frac{\ln\left(\frac{130 - 65}{200 - 65}\right)}{-0.04490} \approx 16.28 \text{ minutes}$$

c. Container 1:

$$t = \frac{\ln\left(\frac{110 - 70}{130 - 70}\right)}{-0.04888} \approx 8.295$$

It will remain between 110º and 130º for about 8.3 minutes.

Container 2:

$$t = \frac{\ln\left(\frac{110 - 60}{130 - 60}\right)}{-0.04118} \approx 8.171$$

It will remain between 110º and 130º for about 8.17 minutes

Container 3:

$$t = \frac{\ln\left(\frac{110 - 65}{130 - 65}\right)}{-0.04490} \approx 8.190$$

It will remain between 110º and 130º for about 8.19 minutes.

d. All three graphs basically lie on top of each other.

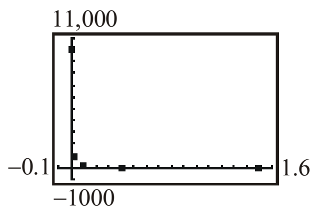
e. Container 1 would be the best. It cools off the quickest but it stays in a warm beverage range the longest.

f. Since all three containers are within seconds of each other in cooling and staying warm, the cost would have an effect. The cheaper one would be the best recommendation.

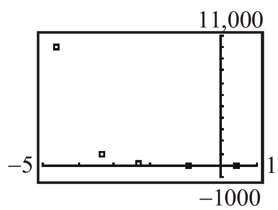
Project 2 (IRC)

Solder Joint Strain, εp	X=ln(εp)	Fatigue Cycles, Nf	Y=ln(Nf)
0.01	–4.605	10,000	9.210
0.035	–3.352	1000	6.908
0.1	–2.303	100	4.605
0.4	–0.916	10	2.303
1.5	0.405	1	0

1.

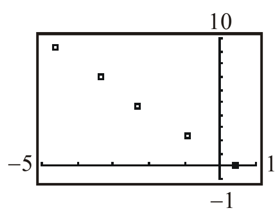


2.



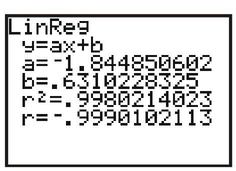
The shape becomes exponential.

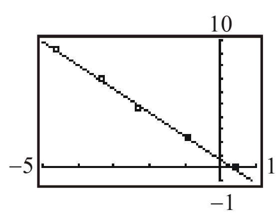
3.



The shape became linear.

4.





$Y = -1.84X + 0.63$

5. $Y = -1.84X + 0.63$

$\ln(Nf) = -1.84\ln(\varepsilon p) + 0.63$

$\ln(Nf) = \ln\left((\varepsilon p)^{-1.84}\right) + \ln(e^{0.63})$

$\ln(Nf) = \ln\left(\left((\varepsilon p)^{-1.84}\right)(e^{0.63})\right)$

$Nf = \left((\varepsilon p)^{-1.84}\right)(e^{0.63})$

$Nf = e^{0.63}(\varepsilon p)^{-1.84}$

6. $Nf = e^{0.63}(0.02)^{-1.84}$

$Nf = 2510.21\ cycles$

$Nf = e^{0.63}(\varepsilon p)^{-1.84}$

$3000 = e^{0.63}(\varepsilon p)^{-1.84}$

$\dfrac{3000}{e^{0.63}} = (\varepsilon p)^{-1.84}$

$\varepsilon p = \left(\dfrac{3000}{e^{0.63}}\right)^{-1/1.84}$

$\varepsilon p = 0.018$

7. $Nf = e^{0.63}(\varepsilon p)^{-1.84}$

$Nf = 1.88(\varepsilon p)^{-1.84}$

$\dfrac{Nf}{1.88} = (\varepsilon p)^{-1.84}$

$\varepsilon p = \left(0.53Nf\right)^{-1/1.84}$

$\varepsilon p = \left(0.53Nf\right)^{-.543}$

$\varepsilon p = 1.41\left(Nf\right)^{-.543}$

$\varepsilon p = 1.41\left(Nf\right)^{-.543}$

$\varepsilon p = 1.41\left(3000\right)^{-.543}$

$\varepsilon p = 0.018$

Project 3 (IRC)

Chart: Answers will vary depending upon the values for the car chosen.

a. Answers will vary.

b. Answers will vary, but in general, they will be in the form $y = ab^x$

c. $A = A_0 b^t$

$A = A_0 e^{rt}$

$\therefore e^r = b$

$r = \ln b$

$A = A_0 e^{(\ln b)t}$

d. A_0 is the purchase price of the vehicle, or a close approximation to it.

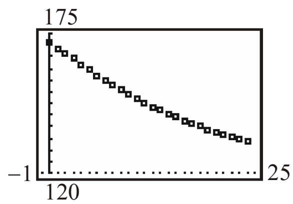
e. Answers will vary.

f. Answers will vary.

g. Answers will vary. However, one might consider how well a car will “hold its value” over time. In other words, one would prefer to purchase a vehicle with a low depreciation rate.

Project 4 (IRC)

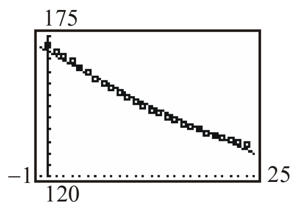
a.



b. $y = ab^x$

$y = 173.29(0.989)^x$

c.



d.
$$110 = 173.29(0.989)^x$$
$$\frac{110}{173.29} = (0.989)^x$$
$$\ln\left(\frac{110}{173.29}\right) = \ln(0.989)^x$$
$$\ln\left(\frac{110}{173.29}\right) = x\ln(0.989)$$
$$x = \frac{\ln\left(\frac{110}{173.29}\right)}{\ln(0.989)} \approx 41$$

41 units of time

e. $u = 70 + (175.69 - 70)e^{kt}$

$u = 70 + 105.69e^{kt}$

Using both equations for u(t), and letting t = 1, we can find k:

$$70 + 105.69e^{k(1)} = 173.29(0.989)^1$$
$$105.69e^k = 101.38$$
$$e^k = \frac{101.38}{105.69}$$
$$k = \ln\left(\frac{101.38}{105.69}\right) \approx -0.042$$
$$u(t) = 70 + 105.69e^{-0.042t}$$

Chapter 4 Cumulative Review

1. The graph represents a function since it passes the Vertical Line Test. The function is not a one-to-one function since the graph fails the Horizontal Line Test.

2. $f(x) = 2x^2 - 3x + 1$

a. $f(3) = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10$

b. $f(-x) = 2(-x)^2 - 3(-x) + 1 = 2x^2 + 3x + 1$

c.
$$\begin{aligned} f(x+h) &= 2(x+h)^2 - 3(x+h) + 1 \\ &= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h + 1 \\ &= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1 \end{aligned}$$

3. $x^2 + y^2 = 1$

a. $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$; $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not on the graph.

b. $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$; $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the graph.

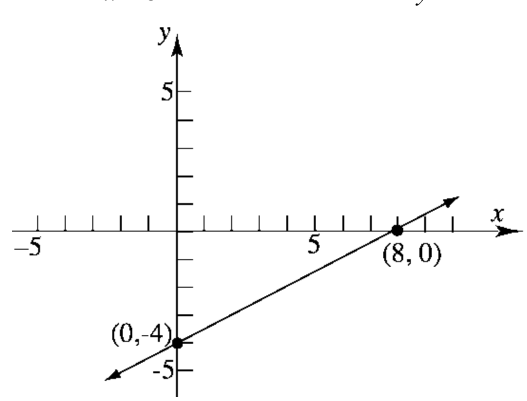
4. $3(x-2) = 4(x+5)$
$$3x - 6 = 4x + 20$$
$$-26 = x$$

The solution set is $\{-26\}$.

5. $2x - 4y = 16$

x-intercept:
$$2x - 4(0) = 16$$
$$2x = 16$$
$$x = 8$$

y-intercept:
$$2(0) - 4y = 16$$
$$-4y = 16$$
$$y = -4$$



6. a. $f(x) = -x^2 + 2x - 3$; $a = -1, b = 2, c = -3$.

Since $a = -1 < 0$, the graph opens down.

The *x*-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{2}{2(-1)} = -\frac{2}{-2} = 1.$$

The *y*-coordinate of the vertex is

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= f(1) \\ &= -1^2 + 2(1) - 3 \\ &= -1 + 2 - 3 \\ &= -2 \end{aligned}$$

Thus, the vertex is $(1, -2)$.

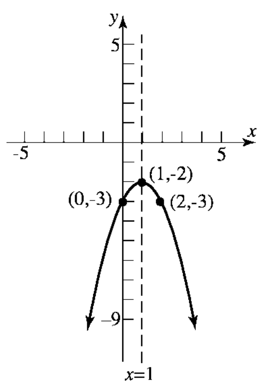
The axis of symmetry is the line $x = 1$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(-1)(-3) = 4 - 12 = -8 < 0$.

The graph has no *x*-intercepts.

The *y*-intercept is $f(0) = -0^2 + 2(0) - 3 = -3$.



b. The graph of $f(x) = -x^2 + 2x - 3$ indicates that $f(x) \le 0$ for all values of *x*.

7. Given that the graph of $f(x) = ax^2 + bx + c$ has vertex $(4, -8)$ and passes through the point $(0, 24)$, we can conclude $-\frac{b}{2a} = 4$, $f(4) = -8$, and $f(0) = 24$.

Notice that

$$\begin{aligned} f(0) &= 24 \\ a(0)^2 + b(0) + c &= 24 \\ c &= 24 \end{aligned}$$

Therefore $f(x) = ax^2 + bx + c = ax^2 + bx + 24$.

Furthermore, $-\frac{b}{2a} = 4$, so that $b = -8a$, and

$$\begin{aligned} f(4) &= -8 \\ a(4)^2 + b(4) + 24 &= -8 \\ 16a + 4b + 24 &= -8 \\ 16a + 4b &= -32 \\ 4a + b &= -8 \end{aligned}$$

Replacing b with $-8a$ in this equation yields

$$\begin{aligned} 4a - 8a &= -8 \\ -4a &= -8 \\ a &= 2 \end{aligned}$$

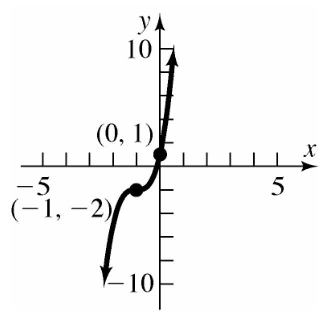
So $b = -8a = -8(2) = -16$.

Therefore, we have the function

$f(x) = 2x^2 - 16x + 24$.

8. $f(x) = 3(x+1)^3 - 2$

Using the graph of $y = x^3$, shift the graph 1 unit to the left, stretch vertically by a factor of 3, and shift 2 units down.



9. $f(x) = x^2 + 2 \qquad g(x) = \dfrac{2}{x-3}$

$$\begin{aligned} f(g(x)) &= f\left(\frac{2}{x-3}\right) \\ &= \left(\frac{2}{x-3}\right)^2 + 2 \\ &= \frac{4}{(x-3)^2} + 2 \end{aligned}$$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

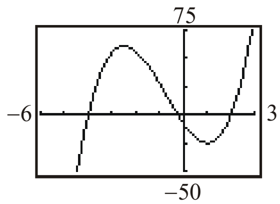
The domain of g is $\{x \mid x \neq 3\}$.

So, the domain of $f(g(x))$ is $\{x \mid x \neq 3\}$.

$$f(g(5)) = \frac{4}{(5-3)^2} + 2 = \frac{4}{2^2} + 2 = \frac{4}{4} + 2 = 3$$

10. $f(x) = 4x^3 + 9x^2 - 30x - 8$

a. The graph of $Y_1 = 4x^3 + 9x^2 - 30x - 8$ appears to indicate zeros at $x = -4$ and $x = 2$.



$$f(-4) = 4(-4)^3 + 9(-4)^2 - 30(-4) - 8$$
$$= -256 + 144 + 120 - 8$$
$$= 0$$
$$f(2) = 4(2)^3 + 9(2)^2 - 30(2) - 8$$
$$= 32 + 36 - 60 - 8$$
$$= 0$$

Therefore, $x = -4$ and $x = 2$ are real zeros for f.

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 4 & 9 & -30 & -8 \\ & & 8 & 34 & 8 \\ \hline & 4 & 17 & 4 & 0 \end{array}$$

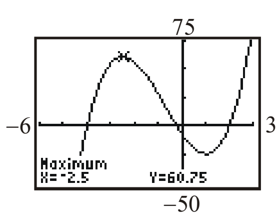
$$f(x) = 4x^3 + 9x^2 - 30x - 8$$
$$= (x-2)(4x^2 + 17x + 4)$$
$$= (x-2)(x+4)(4x+1)$$

Therefore, $x = 2$, $x = -\frac{1}{4}$ and $x = -4$ are real zeros for f.

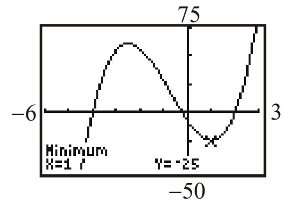
b. f has x-intercepts at $x = 2$, $x = -\frac{1}{4}$ and $x = -4$.

f has y-intercept at $f(0) = 4 \cdot 0^3 + 9 \cdot 0^2 - 30 \cdot 0 - 8 = -8$

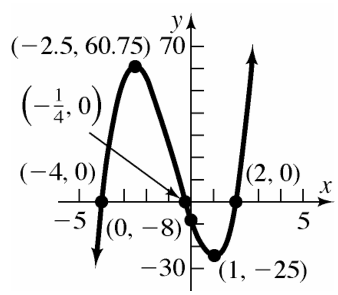
c. Use MAXIMUM to determine that f has a local maximum at the point $(-2.5, 60.75)$.



Use MINIMUM to determine that f has a local minimum at the point $(1, -25)$.



d. Graphing by hand:
The graph of f is above the x-axis for $\left(-4, -\frac{1}{4}\right)$ and $(2, \infty)$.
The graph of f is below the x-axis for $(-\infty, -4)$.

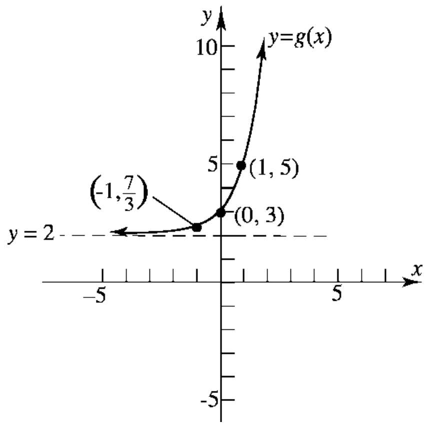


11. a. $g(x) = 3^x + 2$

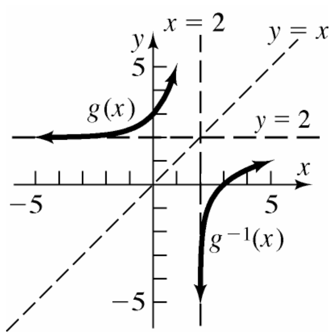
Using the graph of $y = 3^x$, shift up 2 units.
Domain: $(-\infty, \infty)$
Range: $(2, \infty)$
Horizontal Asymptote: $y = 2$

b.
$$g(x) = 3^x + 2$$
$$y = 3^x + 2$$
$$x = 3^y + 2 \quad \text{Inverse}$$
$$x - 2 = 3^y$$
$$y = \log_3(x-2)$$
$$g^{-1}(x) = \log_3(x-2)$$

Domain: $(2, \infty)$
Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 2$



c.



12. $4^{x-3} = 8^{2x}$

$$\left(2^2\right)^{x-3} = \left(2^3\right)^{2x}$$
$$2^{2x-6} = 2^{6x}$$
$$2x-6 = 6x$$
$$-6 = 4x$$
$$x = -\frac{6}{4} = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}\right\}$.

13. $\log_3(x+1) + \log_3(2x-3) = \log_9 9$

$$\log_3\left((x+1)(2x-3)\right) = 1$$
$$(x+1)(2x-3) = 3^1$$
$$2x^2 - x - 3 = 3$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$x = -\frac{3}{2} \text{ or } x = 2$$

Since $\log_3\left(-\frac{3}{2}+1\right) = \log_3\left(-\frac{1}{2}\right)$ is undefined the solution set is $\{2\}$.

14. a. $\log_3(x+2) = 0$

$$x+2 = 3^0$$
$$x+2 = 1$$
$$x = -1$$

The solution set is $\{-1\}$.

b. $\log_3(x+2) > 0$

$$x+2 > 3^0$$
$$x+2 > 1$$
$$x > -1$$

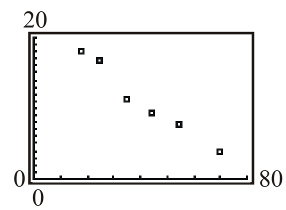
The solution set is $\{x \mid x > -1\}$.

c. $\log_3(x+2) = 3$

$$x+2 = 3^3$$
$$x+2 = 27$$
$$x = 25$$

The solution set is $\{25\}$.

15. a.



b. Answers will vary.

c. Answers will vary.